

Part (1)

Second : Trigonometry

(1) Complete the following table :

The angle Ratio	$42^{\circ} 12'$
Sin	0.3214
Cos	0.5321
Tan	2.0625

(2) Complete the following :

- 1) $46^{\circ} 36' 24'' = \dots\dots\dots$ In degrees.
- 2) $44.125^{\circ} = \dots\dots\dots$ in degrees, minutes, seconds.
- 3) If $\tan \theta = 1.42$ where θ is the measure of an acute angle. Then $\theta = \dots\dots\dots$
- 4) If $\sin \theta = 0.63$ where θ is the measure of an acute angle, then $\theta = \dots\dots\dots$
- 5) If $\sin x = \frac{1}{2}$ where x is an acute angle then $m(\angle x) = \dots\dots\dots$
- 6) If $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$ where x is an acute angle then $m(\angle x) = \dots\dots\dots$
- 7) $\sin 60^{\circ} + \cos 30^{\circ} - \tan 60^{\circ} = \dots\dots\dots$
- 8) $\cos 60^{\circ} + \sin 30^{\circ} - \tan 45^{\circ} = \dots\dots\dots$
- 9) $2 \sin 30^{\circ} \times \cos 60^{\circ} - \tan 45^{\circ} = \dots\dots\dots$
- 10) $\sin^2 30^{\circ} + \cos^2 30^{\circ} = \dots\dots\dots$
- 11) If $\tan (x + 10) = \sqrt{3}$ where x is an acute angle then $m(\angle x) = \dots\dots\dots$
- 12) If $\tan 3x = \sqrt{3}$ where x is an acute angle then $m(\angle x) = \dots\dots\dots$

(3) In the opposite figure:-

ABC is a triangle, $\overline{AD} \perp \overline{BC}$,

AC = 12 cm, BC = 16 cm and $m(\angle C) = 30^\circ$

Complete the following

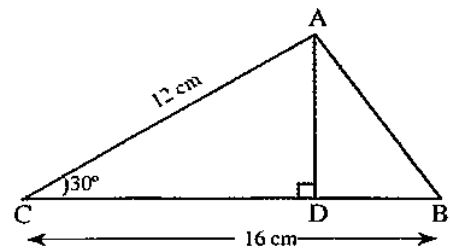
$\therefore \sin 30 = \frac{AD}{\dots\dots\dots}$

$\therefore AD = \dots\dots\dots \times \sin 30^\circ = \dots\dots\dots \text{ cm}$

$\therefore \text{The area of } \Delta ABC = \dots\dots\dots \times AD \times BC$

$\therefore \text{The area of } \Delta ABC = \dots\dots\dots \times \dots\dots\dots \times \dots\dots\dots = \dots\dots\dots \text{ cm}^2$

Can you calculate the height of the triangle which is drawn from the point B on \overleftrightarrow{AC} ? Explain your answer showing the steps of solution



(4) Choose the correct answer form those given:-

1) $4 \cos 30^\circ \tan 60^\circ = \dots\dots\dots$

- a) 3 b) $2\sqrt{3}$ c) 6 d) 12

2) If $\cos 2x = \frac{1}{2}$ where x is an acute angle then $m(\angle x) = \dots\dots\dots$

- a) 15° b) 30° c) 45° d) 60°

3) If $\tan \frac{3x}{2} = 1$ where x is acute angle then $m(\angle x) = \dots\dots\dots$

- a) 10° b) 30° c) 45° d) 60°

4) $2 \tan 45 - \frac{1}{\cos 60^\circ} = \dots\dots\dots$

- a) zero b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1

5) If $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$ where x is an acute angle then $\sin x = \dots\dots\dots$

- a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$

6) In ΔABC :

If $m(\angle A) = 85^\circ$, $\sin B = \cos B$, then $m(\angle C) = \dots\dots\dots$

- a) 30° b) 45° c) 50° d) 60°

(5) Find the value of the following:-

- 1) $(\cos 30^\circ - \cos 60^\circ) (\sin 30^\circ + \sin 60^\circ)$
- 2) $\frac{1}{4} \sin^2 45^\circ \tan^2 60^\circ - \frac{1}{3} \sin^2 60^\circ \tan^2 30^\circ$
- 3) $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$
- 4) $\frac{\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ}$

(6) Prove that:

- 1) $\cos 60^\circ = 2 \cos^2 30^\circ - 1$
- 2) $\tan 60^\circ (1 - \tan^2 30^\circ) = 2 \tan 30^\circ$
- 3) $\tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$
- 4) $\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$
- 5) $\frac{\tan^2 30^\circ \tan 45^\circ \tan^2 60^\circ + \tan 30^\circ \tan 60^\circ}{\sin^2 60^\circ - \tan 45^\circ \sin 30^\circ}$

(7) Find the value of x in each of the following:-

- 1) $x \cos 30^\circ = \tan 60^\circ$
- 2) $x \sin^2 45^\circ = \tan^2 60^\circ$
- 3) $4x = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$
- 4) $x \sin 30^\circ \cos^2 45^\circ = \cos^2 30^\circ$
- 5) $x \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos^2 60^\circ$
- 6) $\tan x = \frac{\sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \sin 45^\circ \sin 60^\circ}$

(8) Find $m(\angle \theta)$ where θ is an acute angle :

- 1) $\sin^2 45^\circ = \cos \theta \tan 30^\circ$
- 2) $2 \sin \theta = \tan^2 60^\circ - 2 \tan 45^\circ$
- 3) $\sin \theta = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 45^\circ$
- 4) $\sin \theta \sin^2 60^\circ = 3 \sin^2 45^\circ \cos^2 45^\circ \cos 60^\circ$
- 5) $\tan \theta = 3 (\sin 30^\circ + \cos 30^\circ) - 4 (\sin^3 60^\circ + \cos^3 60^\circ)$
- 6) $3 \tan^2 \theta = 4 \sin^2 30^\circ + 8 \cos^2 60^\circ$

(9) In the opposite Figure:-

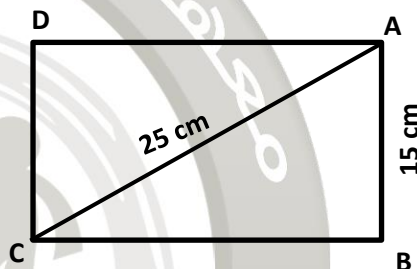
ABCD is a rectangle where $AB = 15\text{cm}$.

$AC = 25\text{cm}$.

Find:

First: $m(\angle ACB)$

Second: The surface area of the rectangle ABCD



(10) In the opposite figure:-

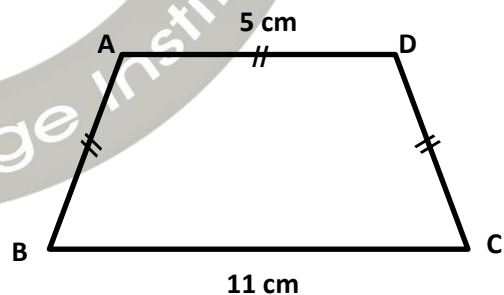
ABCD is an isosceles trapezium

where $AB = AD = DC = 5\text{cm}$.

$BC = 11\text{cm}$, find

First : $m(\angle B)$, $m(\angle A)$

Second: the area of the trapezium ABCD.



Third geometry

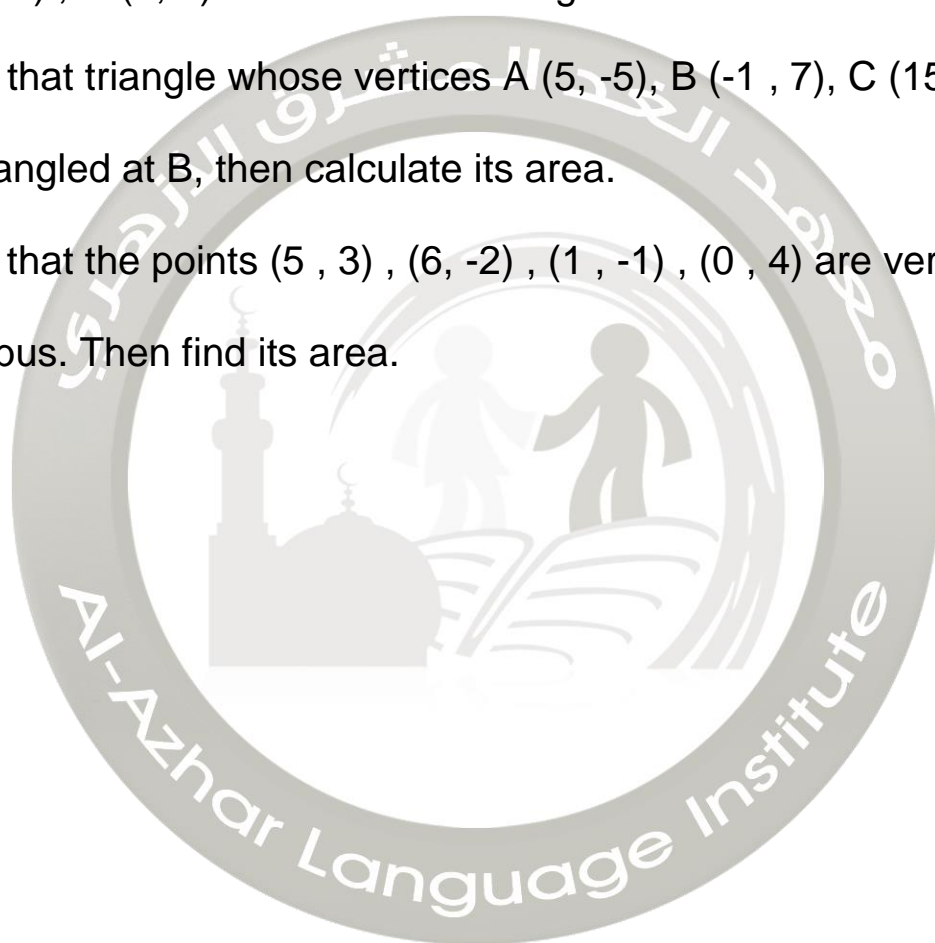
1) Complete each of the following:-

- 1) The distance between the two points $(9, 0)$, $(4, 0)$ is
- 2) The distance between the two points $(0, -11)$, $(0, -5)$ is
- 3) The distance between the points $(4, -3)$ and the origin point is
- 4) The distance between the points $(5, 0)$, $(0, -12)$ is
- 5) The diameter length of the circle whose centre is $(8, 5)$ and passes through the point $(4, 2)$ equals.....
- 6) If the distance between the two points $(a, 0)$ and $(0, 1)$ is one length unit then $a =$
- 7) The distance between the points $(3, 4)$ and the X – axis = length unit.
- 8) In the square ABCD: If $A(2, -5)$, $B(-1, -1)$ then the perimeter of the square is length unit and its area is square unit.

2) Answer the following questions:-

- 1) Find the length of \overline{MN} in each of the following cases:
 - a) $M(2, -1)$, $N(5, 3)$
 - b) $M(-3, -5)$, $N(5, 1)$
 - c) $M(7, -8)$, $N(2, 4)$
 - d) $M(7, -3)$, $N(0, 4)$.
- 2) Prove that the points $A(3, -1)$, $B(-4, 6)$, $C(2, -2)$ which belong to an orthogonal Cartesian co-ordinates plane lie on the circle whose centre $M(-1, 2)$ then find the circumference of the circle.
- 3) Find the value of a in each of the following
 - a) If the distance between the two points $(a, 7)$ and $(-2, 3)$ equals 5.
 - b) If the distance between the two points $(a, 7)$ and $(3a - 1, -5)$ equals 13

- 4) If A (x , 3) , B (3 , 2) , C (5 , 1) and if $AB = BC$ find the value of x.
- 5) If the distance between the point (x , 5) and the point (6 , 1) equals $2\sqrt{5}$ find the value of x.
- 6) Identify the type of the triangle whose vertices are A (-2 , 4) , B (3, -1) , C (4, 5) due to its sides lengths.
- 7) Prove that triangle whose vertices A (5, -5), B (-1 , 7), C (15 , 15) is right angled at B, then calculate its area.
- 8) Prove that the points (5 , 3) , (6, -2) , (1 , -1) , (0 , 4) are vertices of a rhombus. Then find its area.



Part Two

(1) Complete each of the following :

- 1) The mid –point of the line segment joining the two points (2 , 5) and (4 , 3) is the point
- 2) If (2 , 1) is the mid-point of \overline{AB} where A (3, -4) and B (m, 6) then m=
- 3) If the origin point is the mid-point of the line segment \overline{AB} where A (5, -2) then the co-ordinates of B (..... ,))
- 4) If $\overrightarrow{AB} \parallel \overrightarrow{CD}$ and the slope of $\overrightarrow{AB} = 0.75$ then the slope of \overrightarrow{CD} is
- 5) If $\overrightarrow{AB} \perp \overrightarrow{CD}$ and the slope \overrightarrow{AB} is 0.5 then the slope of \overrightarrow{CD} equals
- 6) The slope of the straight line parallel to the straight line passing through the two points (2 , 3) and (-2 , 3) equals
- 7) If the straight line \overrightarrow{AB} is parallel to X-axis where A (8, 3) and B (2, K) then K =.....
- 8) If the straight line \overrightarrow{CD} is parallel to the Y-axis where C(m, 4) and D (-5 ,7) then m=
- 9) ABC is a right angled triangle at B where A (1 , 4) and B(-1, -2) then the slope of $\overrightarrow{BC} =$
- 10) If the straight line which passes through the two points (a , 0) and (0 , 3) and the straight line which makes an angle of measure 30° with the positive direction of the X-axis are perpendicular then a=

11) If $y = m x + c$ represents the equation of a straight line given its slope and the length of the intercepted part of the Y-axis then

- (a) The equation of the straight line when $m = 1$ and $c = 3$ is
- (b) The equation of the straight line when $m = -2$ and $c = 1$ is
- (c) The equation of the straight line when $m = 3$ and $c = 0$ is

12) In the opposite figure:

C (3 , 4) is the mid-point of \overline{AB}

a) $OA = \dots\dots\dots$ length unit.

b) $OB = \dots\dots\dots$ Length unit.

c) The slope of \overrightarrow{AB} is

d) The slope of \overrightarrow{CO} is

e) The slope of \overrightarrow{AO} is

f) The slope of \overrightarrow{BO} is

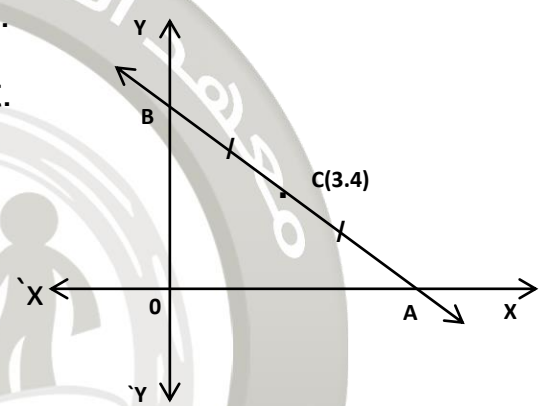
g) C is the centre of the circle which passes through the points
 , ,

h) The area of ΔOAB is square unit.

i) The perimeter of $\Delta OAB = \dots\dots\dots$ unit length.

j) The equation of the \overrightarrow{AB} is

k) The equation \overrightarrow{CO} is



Choose the correct answer from those given:

1- The distance between the point (4 , -3) and the X-axis equals

- a) -3 b) 3 c) 4 d) 5

2- A circle of centre at the origin point and its radius is 2 unit length
which of the following points belongs to the circle?

- a) (1 , 2) b) (-2 , 1) c) ($\sqrt{3}$, 1) d) ($\sqrt{2}$, 1)

3- If (4 , 3) is the mid-point of \overline{AB} where A (3 , 4) then the co-ordinates
of B is

- a) (5 , -2) b) (2 , 5) c) (5 , 2) d) (3.5 , -3.5)

4- The straight line whose equation is $2x - 3y - 6 = 0$ intercepts from
the Y-axis a part of length

- a) -6 b) -2 c) $\frac{2}{3}$ d) 2

5- If the two straight lines $3x - 4y - 3 = 0$ and $ky + 3x - 8 = 0$ are
perpendicular then $k = \dots\dots\dots$

- a) -4 b) -3 c) 3 d) $\frac{9}{4}$

6- If the two straight lines $x + y = 5$ and $kx + 2y = 0$ are parallel then $k =$
.....

- a) -2 b) -1 c) 1 d) 2

7- The area of the triangle bounded by the straight lines $3x - 4y = 12$,
 $x=0$ and $y=0$ is square unit equal

- a) 6 b) 7 c) 12 d) 15

8- \overleftrightarrow{AB} is straight line passing through the two points (2 , 5) and (5 , 2)

which of the following points $\in \overleftrightarrow{AB}$

- a) (1 , 6) b) (2 , 3) c) (0 , 0) d) (3 , -4)

9- The points (0 , 0) , (3 , 0) and (0 , 4)

- a) form an obtuse angles triangle
b) form an octue angled triangle.
c) form a right angled triangle.
d) are collinear.

10- If A (0, 0) , B(5 , 7) and C(5 , h) are the vertices of a right angled triangle at C then h =

- a) zero b) 5 c) 7 d) -5

Answer the following questions:-

1- Find the co-ordinates of the mid-points of \overline{AB} in each of the following:

- a) A (2,4) , B(6,0)
b) A (7 , -5) , B (-3 , 5)
c) A (-3, 6) , B (3, -6)
d) A (7 , -6) , B(-1 , 0)

2- If C is the mid-point of \overline{AB} find x and Y in each of the following cases:

- First : A (1,5), B(3,7), C(x, y)
Second: A (-3, y) , B(9, 11) , C(x, -3)
Third: A (x, -6) , B(9, -11) , C(-3, y)
Fourth: A(x, 3), B(6, y) , C(4, 6)

- 12- Prove that the points A (6, 0), B(2, -4) and C(-4, 2) are vertices of a right angled triangle at B then find the co-ordinates of the point D which makes the figure ABCD a rectangle.
- 13- If the points A (3, 2), B (4, -3), C(-1, -2), D (-2, 3) are vertices of a rhombus find:
- The co-ordinates of the point of intersection of its two diagonals.
 - The area of the rhombus ABCD.
- 14- If A (-1, -1), B(2, 3), C(6, 0), D(3, -4) are four points on an orthogonal Cartesian co-ordinates plane. Prove that \overline{AC} and \overline{BD} bisect each other. What is the name of this figure?
- 15- ABCD is a parallelogram where A (3, -4), B(2, -1), C(-4, 3), find the co-ordinates of point D then find the co-ordinates of point E such that the figure ABCE becomes a trapezium in which $\overline{AE} \parallel \overline{BC}$, $AE=2BC$
- 16-If the straight line L_1 passes through the two points (3, 1) and (2, K), and the straight line L_2 makes with the positive direction of the X-axis and angle of measure 45, Find the value of K if: First : $L_1 \parallel L_2$ second: $L_1 \perp L_2$.
- 17- Using the slope prove that the points A (-1, 3), B (5,1) C(6,4) D(0, 6) are vertices of a rectangle.

Model Answers

Part (1)

Second : Trigonometry

(1) Complete the following table :

The angle Ratio	$42^{\circ} 12'$	$18^{\circ} 44' 51''$	$57^{\circ} 51' 9''$	$64^{\circ} 8' 1''$
Sin	0.6717	0.3214	0.8467	0.8998
Cos	0.7408	0.9469	0.5321	0.4363
Tan	0.9067	0.3394	1.5912	2.0625

(2) Complete the following:

1) 46.6067°

2) $44^{\circ} 7' 30''$

3) $54^{\circ} 50' 45''$

4) $39^{\circ} 3'$

5) 30°

6) $\frac{x}{2} = 30^{\circ} \Rightarrow x = 30 \times 2 = 60^{\circ}$

7) $\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \sqrt{3} = 0$

8) $\frac{1}{2} + \frac{1}{2} - 1 = 0$

9) $2 \times \frac{1}{2} \times \frac{1}{2} - 1 = -\frac{1}{2}$

10) $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

11) $x + 10 = 60 \Rightarrow x = 50^\circ$

12) $3x = 60 \Rightarrow x = 20^\circ$

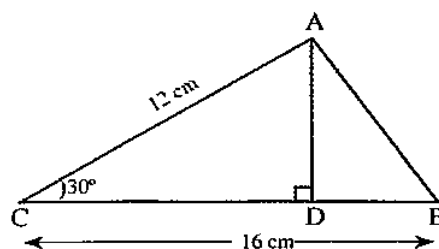
(3) In the opposite figure:-

$\therefore \sin 30^\circ = \frac{AD}{AC} = \frac{AD}{12}$

$\therefore AD = AC \times \sin 30^\circ = 12 \times \frac{1}{2} = 6\text{cm}$

\therefore The area of $\Delta ABC = \frac{1}{2} \times AD \times BC$

\therefore The area of $\Delta ABC = \frac{1}{2} \times 6 \times 16 = 48\text{ cm}^2$



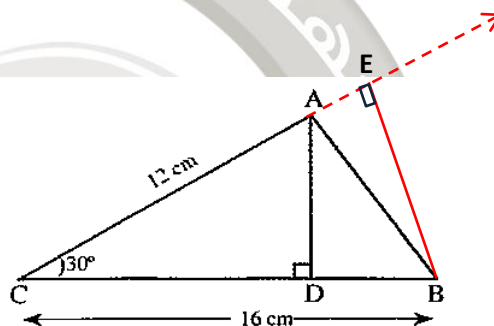
In ΔADC

$\therefore \overline{BE} \perp \overline{AC}$

$\therefore \Delta BEC$ is right angled at E

$\therefore m(\angle C) = 30^\circ, BC = 16\text{ cm}$

$\therefore BE = \frac{1}{2} \times BC = 8\text{ cm}$



(4) Choose :-

- 1) C) 6 2) b (30) 3) $\frac{3x}{2} = 45^\circ \Rightarrow x = 30^\circ$
- 4) Zero 5) $\frac{x}{2} = 30 \rightarrow x = 60^\circ$ d) $\frac{\sqrt{3}}{2}$
- 6) $\therefore \sin B = \cos B \quad \therefore m(\angle B) = 45^\circ \quad m(\angle C) = 50^\circ$

(5) Find the value of the following:-

1) $(\cos 30^\circ - \cos 60^\circ) (\sin 30^\circ + \sin 60^\circ)$
 $= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}-1}{2}\right) \left(\frac{1+\sqrt{3}}{2}\right) = \frac{1}{2}$

2) $\frac{1}{4} \sin^2 45^\circ \tan^2 60^\circ - \frac{1}{3} \sin^2 60^\circ \tan^2 30^\circ$
 $= \frac{1}{4} \times \left(\frac{1}{\sqrt{2}}\right)^2 \times (\sqrt{3})^2 - \frac{1}{3} \times \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{1}{\sqrt{3}}\right)^2$

$$= \frac{1}{4} \times \frac{1}{2} \times 3 - \frac{1}{3} \times \frac{3}{4} \times \frac{1}{3} =$$

$$= \frac{3}{8} - \frac{1}{12} = \frac{7}{24}$$

3) $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

$$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$$

4) $\frac{\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ} = 1$

(6) Prove that:

1) $\cos 60^\circ = 2 \cos^2 30^\circ - 1$

$$\cos 60^\circ = \frac{1}{2}$$

$$2 \cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

2) $\tan 60^\circ (1 - \tan^2 30^\circ) = 2 \tan 30^\circ$

$$\tan 60^\circ (1 - \tan^2 30^\circ) = \sqrt{3} \left(1 - \left(\frac{1}{\sqrt{3}}\right)^2\right)$$

$$2 \tan 30^\circ = 2 \times \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \sqrt{3}$$

$$= \sqrt{3} \left(1 - \frac{1}{3}\right) = \sqrt{3} \left(\frac{2}{3}\right) = \frac{2}{3} \sqrt{3}$$

3) $\tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$

$$\tan^2 60^\circ - \tan^2 45^\circ = (\sqrt{3})^2 - (1)^2$$

$$= 3 - 1 = 2$$

$$4 \sin 30^\circ = 4 \times \frac{1}{2} = 2$$

4) $\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

$$\tan 60^\circ = \sqrt{3}$$

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2x \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$$

5)
$$\frac{\tan^2 30^\circ \tan 45^\circ \tan^2 60^\circ + \tan 30^\circ \tan 60^\circ}{\sin^2 60^\circ - \tan 45^\circ \sin 30^\circ}$$

$$\frac{\left[\left(\frac{1}{\sqrt{3}}\right)^2 \times (1) \times (\sqrt{3})^2\right] + \left[\frac{1}{\sqrt{3}} \times \sqrt{3}\right]}{\left[\left(\frac{\sqrt{3}}{2}\right)^2 - (1 \times \frac{1}{2})\right]}$$

$$= \frac{1+1}{\left[\frac{3}{4} - \frac{1}{2}\right]} = \frac{2}{\frac{1}{4}} = 8$$

(7) Find the value of x in each of the following:-

1) $x \cos 30^\circ = \tan 60^\circ$

$$x = \frac{\tan 60^\circ}{\cos 30^\circ} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = \sqrt{3} \times \frac{2}{\sqrt{3}} = 2$$

2) $x \sin^2 45^\circ = \tan^2 60^\circ$

$$x = \frac{\tan^2 60^\circ}{\sin^2 45^\circ} = \frac{(\sqrt{3})^2}{\left(\frac{\sqrt{2}}{2}\right)^2} = 3 \div \frac{1}{2} = 3 \times 2 = 6$$

3) $4x = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

$$4x = \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{\sqrt{3}}{3}\right)^2 (1)^2 = \frac{3}{4} \times \frac{3}{9} \times 1$$

$$4x = \frac{1}{4} \Rightarrow x = \frac{1}{8}$$

4) $x \sin 30^\circ \cos^2 45^\circ = \cos^2 30^\circ$

$$x = \frac{\cos^2 30^\circ}{\sin 30^\circ \cos^2 45^\circ} = \frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times 4 = 3$$

5) $x \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos^2 60^\circ$

$$x = \frac{\tan^2 45^\circ - \cos^2 60^\circ}{\sin 45^\circ \cos 45^\circ \tan 60^\circ}$$

$$= \frac{1 - \frac{1}{4}}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \times \sqrt{3}} = \frac{3}{4} \times \frac{2}{\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

$$x = \frac{\sqrt{3}}{2}$$

$$6) \tan x = \frac{\sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \sin 45^\circ \sin 60^\circ}$$

$$\tan x = \frac{\frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}} = 1$$

$$\therefore x = 45^\circ$$

(8) Find m ($\angle \theta$) where θ is an acute angle:

$$1) \sin^2 45^\circ = \cos \theta \tan 30^\circ$$

$$\cos \theta = \frac{\sin^2 45^\circ}{\tan 30^\circ} = \frac{1}{2} \times \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

$$2) 2 \sin \theta = \tan^2 60^\circ - 2 \tan 45^\circ$$

$$2 \sin \theta = 3 - 2 = 1$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$$3) \sin \theta = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$\sin \theta = \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2} \times \frac{1}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\theta = 75^\circ$$

$$4) \sin \theta \sin^2 60^\circ = 3 \sin^2 45^\circ \cos^2 45^\circ \cos 60^\circ$$

$$\sin \theta = \frac{3 \sin^2 45^\circ \cos^2 45^\circ \cos 60^\circ}{\sin^2} = \frac{3 \times \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{1}{2}}{\frac{3}{4}}$$

$$\sin \theta = \frac{3\sqrt{2}}{8} \times \frac{4}{3} = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

$$5) \tan \theta = 3 (\sin 30^\circ + \cos 30^\circ) - 4 (\sin^3 60^\circ + \cos^3 60^\circ)$$

$$\tan \theta = 3 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) - 4 \left(\left(\frac{\sqrt{3}}{2} \right)^3 + \left(\frac{1}{2} \right)^3 \right)$$

$$\begin{aligned} \tan \theta &= \frac{3+3\sqrt{3}}{2} - 4 \left(\frac{3\sqrt{3}}{8} + \frac{1}{8} \right) \\ &= \frac{3+3\sqrt{3}}{2} - \frac{3\sqrt{3}+1}{2} = \frac{2}{2} = 1 \end{aligned}$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

$$6) 3 \tan^2 \theta = 4 \sin^2 30^\circ + 8 \cos^2 60^\circ$$

$$3 \tan^2 \theta = 4 \times \frac{1}{4} + 8 \times \frac{1}{4} = 1 + 2 = 3$$

$$\tan^2 \theta = \frac{3}{3} = 1$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

(9) In the opposite Figure:-

In ΔABC

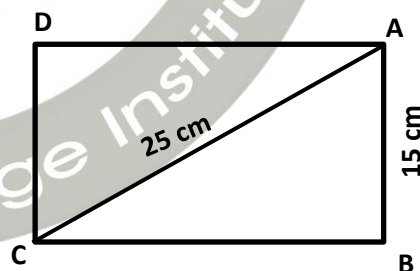
$$\therefore m(\angle B) = 90^\circ$$

$$\therefore \sin(\angle ACB) = \frac{AB}{AC} = \frac{15}{25} = \frac{3}{5}$$

$$m(\angle ACB) = 38^\circ 52' 12''$$

$$BC = \sqrt{AC^2 - AB^2} = 20$$

$$\text{Area of rectangle} = L \times W = 20 \times 15 = 300 \text{cm}^2$$



(10) In the opposite figure:-

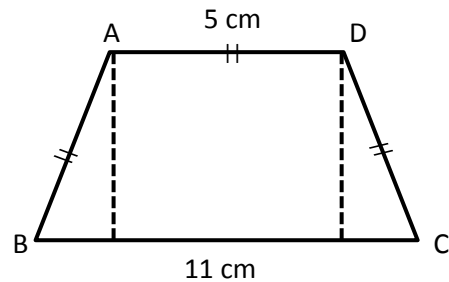
Construction: Draw

$$\vec{AE} \perp \vec{BC} . \vec{DF} \perp \vec{BC}$$

$$\therefore AD = AB = DC = 5\text{cm}$$

$$\therefore EF = 5\text{cm} , BE + CF = 11 - 5 = 6 \text{ cm.}$$

$$\therefore BE = FC = 3\text{cm.}$$



In ΔAEB

$$\therefore m(\angle AEB) = 90^\circ , AB = 5\text{cm} , BE = 3 \text{ cm.}$$

$$\therefore \text{Cos } \angle (B) = \frac{BE}{AB} = \frac{3}{5}$$

$$\therefore m \angle (B) = 53^\circ 7' 48''$$

$$m(\angle BAE) = 180^\circ - (90 + 53^\circ 7' 48'') = 36^\circ 52' 12''$$

$$AE = 4\text{cm}$$

"Pythagoras"

$$\begin{aligned} \text{Area of trapezium} &= \frac{B_1 + B_2}{2} \times H \\ &= \frac{5+11}{2} \times 4 = 8 \times 4 = 32 \text{ cm}^2 \end{aligned}$$

Third geometry

1) Complete each of the following:-

$$\begin{aligned} 1) \quad &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \\ &= \sqrt{(9 - 4)^2 + (0 - 0)^2} = \sqrt{25} = 5 \text{ length unit.} \end{aligned}$$

$$2) D = \sqrt{(0 - 0)^2 + (-11 + 5)^2} = \sqrt{36} = 6 \text{ length unit.}$$

$$3) D = \sqrt{(4 - 0)^2 + (-3 - 0)^2} = \sqrt{16 + 9} = 5 \text{ length unit.}$$

$$4) D = \sqrt{(5 - 0)^2 + (0 + 12)^2} = \sqrt{25 + 144} = 13 \text{ length unit.}$$

$$5) r = \sqrt{(8 - 4)^2 + (5 - 2)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ length unit.}$$

$$\therefore \text{Diameter} = 2r = 10 \text{ length unit.}$$

$$6) D = \sqrt{(a - 0)^2 + (0 - 1)^2} = 1$$

$$\sqrt{a^2 + 1} = 1$$

$$a^2 + 1 = 1^2 = 1$$

$$a^2 = 1 - 1 = 0 \Rightarrow a = 0$$

$$7) |-4| = 4 \text{ length unit.}$$

$$8) AB = \sqrt{(2 + 1)^2 + (-5 + 1)^2} = \sqrt{9 + 16}$$

$$AB = \sqrt{25} = 5 \text{ length unit.}$$

$$P. \text{ of square} = \text{side length} \times 4 = 4 \times 5 = 20 \text{ length unit.}$$

$$\text{area} = S^2 = 5^2 = 25 \text{ squared length unit.}$$

2) Answer the following questions:-

$$1) a) MN = \sqrt{(5 - 2)^2 + (3 + 1)^2} = \sqrt{9 + 16} = 5 \text{ length unit.}$$

$$b) MN = \sqrt{(5 + 3)^2 + (1 + 5)^2} = \sqrt{64 + 36} = 10 \text{ length unit.}$$

$$c) MN = \sqrt{(2 - 7)^2 + (4 + 8)^2} = \sqrt{25 + 144} = 13 \text{ length unit.}$$

$$d) MN = \sqrt{(7 + 0)^2 + (-3 - 4)^2} = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$$

$$2) D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MA = \sqrt{(-1 - 3)^2 + (2 + 1)^2} = \sqrt{16 + 9} = 5 \text{ length unit.}$$

$$MB = \sqrt{(-1 + 4)^2 + (2 + 2)^2} = \sqrt{9 + 16} = 5 \text{ length unit.}$$

$$MC = \sqrt{(-1 - 2)^2 + (2 + 2)^2} = \sqrt{9 + 16} = 5 \text{ length unit.}$$

$$\therefore MA = MB = MC = r$$

$\therefore A, B,$ and C lie on the circle M

$$\text{circumference} = 2\pi r = 2 \times 3.14 \times 5 = 31.4 \text{ length unit.}$$

$$3) D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(5)^2 = (\sqrt{(a + 2)^2 + (7 - 3)^2})^2$$

$$25 = (a + 2)^2 + 16$$

$$(a + 2)^2 = 25 - 16 = 9$$

$$\sqrt{(a + 2)^2} = \pm \sqrt{9}$$

$$a + 2 = \pm 3$$

$$a + 2 = 3 \quad \text{or} \quad a + 2 = -3$$

$$a = 1 \quad \text{or} \quad a = -5$$

$$(b) 13 = \sqrt{(3a - 1 - a)^2 + (-5 - 7)^2}$$

$$(13)^2 = \sqrt{(2a - 1)^2 + 144}$$

$$169 = (2a - 1)^2 + 144$$

$$(2a - 1)^2 = 169 - 144 = 25$$

$$\sqrt{(2a - 1)^2} = \pm \sqrt{25}$$

$$2a - 1 = \pm 5$$

$$\therefore 2a - 1 = 5 \quad \text{or} \quad 2a - 1 = -5$$

$$2a = 6 \rightarrow a = 3 \quad \text{or} \quad 2a = -4 \rightarrow a = -2$$

4) ∴ AB = BC

$$\therefore \sqrt{(x - 3)^2 + (3 - 2)^2} = \sqrt{(3 - 5)^2 + (2 - 1)^2}$$

$$(\sqrt{(x - 3)^2 + 1})^2 = \sqrt{4 + 1} = (\sqrt{5})^2$$

$$(x - 3)^2 + 1 = 5$$

$$(x - 3)^2 = 4$$

$$\sqrt{(x - 3)^2} = \pm \sqrt{4}$$

$$x - 3 = \pm 2$$

$$x - 3 = 2$$

or

$$x - 3 = -2$$

$$x = 5$$

or

$$x = 1$$

5) D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$2\sqrt{5} = \sqrt{(x - 6)^2 + (5 - 1)^2}$$

$$(2\sqrt{5})^2 = \sqrt{(x - 6)^2 + 16}^2$$

$$20 = (x - 6)^2 + 16$$

$$(x - 6)^2 = 20 - 16 = 4$$

$$\sqrt{(x - 6)^2} = \pm \sqrt{4}$$

$$x - 6 = \pm 2$$

$$x - 6 = 2$$

or

$$x - 6 = -2$$

$$x = 8$$

or

$$x = 4$$

$$6) AB = \sqrt{(3 + 2)^2 + (-1 - 4)^2} = \sqrt{25 + 25} = \sqrt{50}$$

$$BC = \sqrt{(4 - 3)^2 + (5 + 1)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$AC = \sqrt{(4 + 2)^2 + (5 - 4)^2} = \sqrt{36 + 1} = \sqrt{37}$$

$$\therefore AC = BC = \sqrt{37}$$

$\therefore \Delta ABC$ is an isosceles Δ

$$7) AB = \sqrt{(5 + 1)^2 + (-5 - 7)^2} = \sqrt{36 + 144} = 6\sqrt{5}$$

$$BC = \sqrt{(15 + 1)^2 + (15 - 7)^2} = \sqrt{256 + 64} = \sqrt{320} = 8\sqrt{5}$$

$$CA = \sqrt{(15 - 5)^2 + (15 + 5)^2} = \sqrt{100 + 400} = 10\sqrt{5}$$

$$AC^2 = (10\sqrt{5})^2 = 500$$

$$AB^2 + BC^2 = 180 + 320 = 500$$

$$\therefore AC^2 = AB^2 + BC^2$$

$\therefore ABC$ is right-angled Δ at B

8) A (5 , 3) , B (6 , -2) , C (1 , -1) , D (0 , 4)

$$AB = \sqrt{(6 - 5)^2 + (-2 - 3)^2} = \sqrt{1 + 25} = \sqrt{26}$$

$$BC = \sqrt{(6 - 1)^2 + (-2 + 1)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$CD = \sqrt{(1 - 0)^2 + (-1 - 4)^2} = \sqrt{1 + 25} = \sqrt{26}$$

$$DA = \sqrt{(5 - 0)^2 + (3 - 4)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$\therefore AB = BC = CD = DA.$$

\therefore A, B, C, and D are vertices of Rhombus.

$$AC = \sqrt{(5 - 1)^2 + (3 + 1)^2} = \sqrt{16 + 16} = \sqrt{32}$$

$$BD = \sqrt{(6 - 0)^2 + (-2 - 4)^2} = \sqrt{36 + 36} = \sqrt{72}$$

$$\text{Area of the rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times \sqrt{32} \times \sqrt{72} = 24 \text{ (u.l.)}^2$$

Part Two

(1) Complete each of the following :

$$\begin{aligned}
 1- \text{midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \\
 &= \left(\frac{2+4}{2}, \frac{5+3}{2} \right) = (3, 4)
 \end{aligned}$$

$$\begin{aligned}
 2- (2, 1) &= \left(\frac{m+3}{2}, \frac{6-4}{2} \right) \\
 \frac{m+3}{2} &= 2 \quad \rightarrow m + 3 = 4 \\
 \rightarrow m &= 4 - 3 = +1
 \end{aligned}$$

3- B (-5 , 2) because the origin point is the mid point.
Or we can use the rule of mid point.

$$4- 0.75 \rightarrow m_1 = m_2$$

$$5- m_1 = \frac{-1}{m_2} \rightarrow \frac{-1}{0.5} = -2$$

$$6- m_1 = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{2+2} = 0$$

7- If $\overleftrightarrow{AB} \parallel x\text{-axis}$
 \therefore its slope = 0

$$\frac{K-3}{2-8} = 0 \Rightarrow K - 3 = 0 \Rightarrow K = 3$$

8- $\overleftrightarrow{CD} \parallel y\text{-axis}$

The slope is undefined where the denominator equal to zero.

$$\text{Slope } \frac{7-4}{-5-m} \Rightarrow -5 - m = 0 \Rightarrow m = -5$$

9- m_1 of $\overleftrightarrow{AB} = \frac{4+2}{1+1} = \frac{6}{2} = 3$

m_2 of $\overleftrightarrow{BC} = -\frac{1}{3}$

because $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$, then $m_2 = \frac{-1}{m_1}$

10- $m_1 = \frac{3-0}{0-a} = -\frac{3}{a}$

$m_2 = \tan 30 = \frac{\sqrt{3}}{3}$

$m_1 \times m_2 = -1$

$-\frac{3}{a} \times \frac{\sqrt{3}}{3} = -1$

$\frac{\sqrt{3}}{a} = 1$

$\therefore a = \sqrt{3}$

11- a) $y = x + 3$

b) $y = -2x + 1$

c) $Y = 3x$

12- A (x , 0) , B(0, y)

\therefore C (3 , 4) is midpoint

$\therefore \left(\frac{x+0}{2}, \frac{y+0}{2}\right) = (3, 4)$

$\frac{x}{2} = 3 \rightarrow x = 6$

$\frac{y}{2} = 4 \rightarrow y = 8$

a) OA = 6 L.U.

b) OB = 8 L.U.

c) $m = \frac{8-0}{0-6} = \frac{8}{-6} = \frac{-4}{3}$

d) $m = \frac{4-0}{3-0} = \frac{4}{3}$

e) $m = \frac{0-0}{0-0} = 0$

f) $m = \frac{8-0}{0-0} = \text{undefined}$

g) A, B, O

h) $\frac{1}{2} \times 6 \times 8 = 24$ square unit

i) $AB = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = 10$

Perimeter = $6 + 10 + 8 = 24$ L.U.

j) m of $\overrightarrow{AB} = -\frac{4}{3}$, $C = 8$

$y = -\frac{4}{3}x + 8$

k) m of $\overrightarrow{CO} = \frac{4}{3}$, $C = 0$

$y = \frac{4}{3}x$

Second : Choose

1) b

4) d

7) a

10) a

2) c

5) d

8) a

3) c

6) d

9) c

Answer the following questions

1- Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

a) $M = \left(\frac{2+6}{2}, \frac{4+0}{2}\right) = (4, 2)$

b) $M = \left(\frac{7-3}{2}, \frac{-5+5}{2}\right) = (2, 0)$

c) $M = \left(\frac{-3+3}{2}, \frac{-6+0}{2}\right) = (0, -3)$

d) $\left(\frac{7-1}{2}, \frac{-6+0}{2}\right) = (3, -3)$

2- a) $(x, y) = \left(\frac{1+3}{2}, \frac{5+7}{2}\right) = (2, 6)$

$X=2, y = 6$

b) $(x, -3) = \left(\frac{-3+9}{2}, \frac{y+11}{2}\right) = \left(3, \frac{y+11}{2}\right)$

$x = 3, \frac{y+11}{2} = -3 \rightarrow y + 11 = -6 \rightarrow y = -17$

c) $(-3, y) = \left(\frac{x+9}{2}, \frac{-6-11}{2}\right) = \left(\frac{x+9}{2}, -8.5\right)$

$\frac{x+9}{2} = -3 \rightarrow x + 9 = -6 \rightarrow x = -15, y = -8.5$

d) $(4, 6) = \left(\frac{x+6}{2}, \frac{3+y}{2}\right)$

$\frac{x+6}{2} = 4 \rightarrow x + 6 = 8 \rightarrow x = 2$

$\frac{3+y}{2} = 6 \rightarrow 3 + y = 12 \rightarrow y = 9$

3- a) $m = \tan 30^\circ = \frac{\sqrt{3}}{3}$

b) $m = \tan 45^\circ = 1$

c) $m = \tan 60^\circ = \sqrt{3}$

4- a) $\theta = \tan^{-1} 0.3673 = 20^\circ 10' 6''$

b) $\theta = \text{shift } \tan^{-1} 1.0246 = 45^\circ 41' 46''$

c) $\theta = \tan^{-1} 3.1648 = 72^\circ 27' 53''$

5- ∴ the points are collinear.

∴ the slope of \overleftrightarrow{AB} = slope of \overleftrightarrow{BC} where A (0,1), B(a,3), C(2,5)

m_1 of $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{a-0} = \frac{2}{a}$

m_2 of $\overleftrightarrow{BC} = \frac{3-5}{a-2} = \frac{-2}{a-2}$

$\frac{2}{a} = \frac{-2}{a-2}$

$$-2a = 2a - 4 \rightarrow 2a + 2a = 4$$

$$4a = 4 \rightarrow a = 1$$

6- m_1 of $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 + 3}{-1 - 0} = -8$

$$m_2$$
 of $\overleftrightarrow{BC} = \frac{-3 - 1}{0 - 2} = \frac{-4}{-2} = 2$

$\therefore m_1 \neq m_2 \quad \therefore A, B$ and are not collinear.

Second : m_1 of $\overleftrightarrow{AB} = \frac{3 - 1}{2 + 2} = \frac{2}{4} = \frac{1}{2}$, m_2 of $\overleftrightarrow{BC} = \frac{4 - 3}{4 - 2} = \frac{1}{2}$

$\therefore m_1 = m_2$, B is a common point.

$\therefore A, B$, and C are collinear.

7- Slope of $\overleftrightarrow{AB} = \frac{3 - 5}{3 + 2} = \frac{-2}{5}$

$$\text{Slope of } \overleftrightarrow{BC} = \frac{2 - 3}{-4 - 3} = \frac{1}{7}$$

\therefore slope of $\overleftrightarrow{AB} \neq$ slope of \overleftrightarrow{BC}

$\therefore A, B$, and C are not collinear

$$\text{Slope of } \overleftrightarrow{CD} = \frac{2 - 4}{-4 + 9} = \frac{-2}{5}$$

$$\text{Slope of } \overleftrightarrow{DA} = \frac{4 - 5}{-9 + 2} = \frac{-1}{-7} = \frac{1}{7}$$

\therefore slope of $\overleftrightarrow{BC} =$ slope of \overleftrightarrow{DA}

\therefore slope of $\overleftrightarrow{AB} =$ slope of \overleftrightarrow{DA}

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{DA}$

$\therefore ABCD$ is a parallelogram.

8- midpoint of $\overline{BC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$$D = \left(\frac{3 + 1}{2}, \frac{7 - 3}{2} \right) = (2, 2)$$

A(5, -6), D(2, 2)

$$m \text{ of } \overleftrightarrow{AD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2+6}{2-5} = \frac{-8}{3}$$

∴ A (5, -6) lies on the s.line

∴ A satisfies the equation of s.L.

$$\therefore y = \frac{-8}{3}x + C \quad \text{at A (5, -6)}$$

$$-6 = \frac{-8}{3} \times 5 + c$$

$$-6 = \frac{-40}{3} + c$$

$$C = -6 + \frac{40}{3} = -\frac{18}{3} + \frac{40}{3} = \frac{22}{3}$$

$$y = \frac{-8}{3}x + \frac{22}{3}$$

Or

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{-8}{3} = \frac{y+6}{x-5}$$

$$-8x + 40 = 3y + 18$$

$$3y = -8x + 22$$

$$y = \frac{-8}{3}x + \frac{22}{3}$$

9- $m_1 = \frac{-\text{co efficient of } x}{\text{co efficient of } y} = \frac{-1}{2}$

or $x+2y-7=0$

$$2y = -x + 7$$

$$y = \frac{-1}{2}x + \frac{7}{2}$$

$$\therefore m_1 = -\frac{1}{2}$$

∴ the two st. lines are parallel

$$\therefore m_1 = m_2 = -\frac{1}{2}$$

$$\therefore y = mx + c$$

$$y = -\frac{1}{2}x + C \quad \text{at (3, -5)}$$

$$-5 = -\frac{1}{2} \times 3 + C$$

$$-5 = -\frac{3}{2} + c \rightarrow c = -5 + \frac{3}{2} = \frac{-7}{2} \Rightarrow y = -\frac{1}{2}x - \frac{7}{2}$$

10- ∴ the st. line cuts x-axis at 4

∴ (4, 0) ∈ the st. line

∴ The s. line cuts y axis at 9

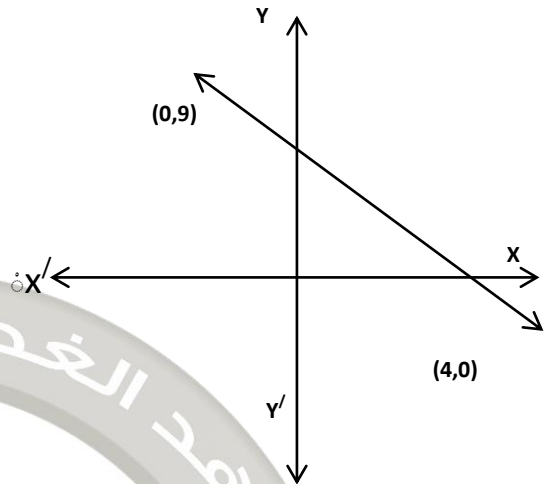
∴ (0, 9) ∈ the st. line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 0}{0 - 4} = \frac{-9}{4}$$

$$y = mx + c \rightarrow y = -\frac{9}{4}x + c$$

$$\text{at } (0, 9) \rightarrow c = 9$$

$$y = \frac{-9}{4}x + 9$$



11- let C is the midpoint of \overline{AB}

$$C = \left(\frac{1+9}{2}, \frac{-6+2}{2} \right) = (5, -2)$$

Let D is the midpoint of \overline{CB}

$$D = \left(\frac{5+9}{2}, \frac{-2+2}{2} \right) = (7, 0)$$

Let E is the mid-point of of \overline{AC}

$$E = \left(\frac{5+1}{2}, \frac{-2-6}{2} \right) = (3, -4)$$

12- $AB = \sqrt{(6 - 2)^2 + (0 + 4)^2} = \sqrt{32}$

$$BC = \sqrt{(2 + 4)^2 + (-4 - 2)^2} = \sqrt{72}$$

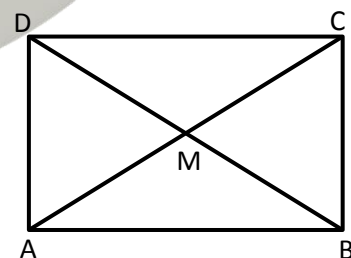
$$CA = \sqrt{(6 + 4)^2 + (0 - 2)^2} = \sqrt{104}$$

$$\therefore CA^2 = 104$$

$$\therefore AB^2 + BC^2 = 32 + 72 = 104$$

$$\therefore CA^2 = AB^2 + BC^2 = 104$$

∴ ABC is right angles Δ at B



If M is the point of inter section of the two diagonals \overline{AC} and \overline{BD}

∴ M is a mid-point of \overline{AC} and \overline{BD}

$$M = \left(\frac{6-4}{2}, \frac{0+2}{2} \right) = (1, 1)$$

Let D (x , y)

$$(1, 1) = \left(\frac{x+2}{2}, \frac{y-4}{2} \right)$$

$$\frac{x+2}{2} = 1 \rightarrow x + 2 = 2 \rightarrow x=0$$

$$\frac{y-4}{2} = 1 \rightarrow y - 4 = 2 \rightarrow y=6$$

$$D = (0, 6)$$

13- let m is the point of intersection of its two diagonals.

∴ m is the mid point of \overline{AC}

$$m = \left(\frac{3-1}{2}, \frac{2-2}{2} \right) = (1, 0)$$

$$AC = \sqrt{(3+1)^2 + (2+2)^2} = \sqrt{16+16} = \sqrt{32} \text{ l.u.}$$

$$BD = \sqrt{(4+2)^2 + (-3-3)^2} = \sqrt{36+36} = \sqrt{72} \text{ l.u.}$$

$$\text{Area of the rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times \sqrt{32} \times \sqrt{72} = 24 \text{ (lu)}^2$$

14- midpoint of $\overline{AC} = \left(\frac{-1+6}{2}, \frac{-1+0}{2} \right) = (2.5, -0.5)$

midpoint of $\overline{BD} = \left(\frac{2+3}{2}, \frac{3-4}{2} \right) = (2.5, -0.5)$

∴ The point of \overline{AC} = the mid point \overline{BD}

∴ \overline{AC} and \overline{BD} bisect each other.

$$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16} = 5 \text{ l.u.}$$

$$BC = \sqrt{(6-2)^2 + (0-3)^2} = \sqrt{16+9} = 5 \text{ l.u.}$$

$$CD = \sqrt{(6-3)^2 + (0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ l.u.}$$

$$DA = \sqrt{(3+1)^2 + (-4+1)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ I.u.}$$

$$AC = \sqrt{(6+1)^2 + (0+1)^2} = \sqrt{49+16} = \sqrt{50} \text{ I.u.}$$

$$BD = \sqrt{(3-2)^2 + (-4-3)^2} = \sqrt{1+49} = \sqrt{50} \text{ I.u.}$$

$$\therefore AB = BC = CD = DA$$

$$\therefore AC = BD$$

\therefore ABCD is a square .

15- M is the midpoint of \overline{AC} and \overline{BD}

$$m(x_1, Y_1) = \left(\frac{-4+3}{2}, \frac{3-4}{2} \right) = \left(-\frac{1}{2}, -\frac{1}{2} \right)$$

$$\left(-\frac{1}{2}, -\frac{1}{2} \right) = \left(\frac{x+2}{2}, \frac{y-1}{2} \right)$$

$$\frac{x+2}{2} = -\frac{1}{2} \rightarrow x+2 = -1 \rightarrow x = -3$$

$$\frac{y-1}{2} = -\frac{1}{2} \rightarrow y-1 = -1 \rightarrow y = 0$$

$$D (-3, 0)$$

\therefore ABCD is a parallelogram.

$$\therefore AE = 2BC$$

$$\therefore AD = DE = BC$$

\therefore D is mid point of \overline{AE}

$$(-3, 0) = \left(\frac{3+x}{2}, \frac{4+y}{2} \right)$$

$$\frac{3+x}{2} = -3 \rightarrow 3+x = -6 \rightarrow x = -9$$

$$\frac{-4+y}{2} = 0 \rightarrow -4+Y = 0 \rightarrow Y = 4$$

$$E (-9, 4)$$

16- First:

$$m_1 \frac{y_2 - y_1}{x_2 - x_1} = \frac{k-1}{2-3} = \frac{k-1}{-1}$$

$$m_2 \tan 45^\circ = 1$$

$$\therefore L_1 \parallel L_2$$

$$\therefore \frac{k-1}{-1} = 1 \Rightarrow k - 1 = -1 \rightarrow k = 0$$

Second : $L_1 \perp L_2$

$$m_1 m_2 = -1$$

$$\frac{k-1}{-1} \times 1 = -1$$

$$K - 1 = 1 \rightarrow k = 2$$

17- using the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_1 \text{ of } \overline{AB} = \frac{1-3}{5+1} = \frac{-2}{6} = -\frac{1}{3}$$

$$m_2 \text{ of } \overline{BC} = \frac{4-1}{6-5} = \frac{3}{1} = 3$$

$$m_3 \text{ of } \overline{CD} = \frac{6-4}{0-6} = \frac{-2}{6} = -\frac{1}{3}$$

$$m_4 \text{ of } \overline{DA} = \frac{6-3}{0+1} = \frac{3}{1} = 3$$

$$\therefore \text{slope of } \overline{AB} = \text{slope of } \overline{CD}$$

$$, \text{slope of } \overline{BC} = \text{slope of } \overline{DA}$$

$$\therefore \overline{AB} \parallel \overline{CD} , \overline{BC} \parallel \overline{DA}$$

$$\therefore \text{slope of } \overline{AB} \times \text{slope of } \overline{BC} = \frac{-1}{3} \times 3 = -1$$

$$\therefore \overline{AB} \perp \overline{BC}$$

\therefore ABCD is a rectangle.