

Part (1)

Complete each of the following:-

- 1) The point (5 , -3) lies in quadrant.
- 2) If $(x + 5 , 8) = (1 , 6y + x)$ then $x = \dots\dots\dots$, $y = \dots\dots\dots$
- 3) If $n(X) = 5$, $n(X \times Y) = 15$ then $n(Y) = \dots\dots\dots$
- 4) The point (4 , 0) lies on axis.
- 5) If $(5 , x-7) = (y + 1 , -5)$ then $x + y = \dots\dots\dots$
- 6) If $X \times Y = \{(1 , 5) , (1 , 7) , (2 , 5) , (2 , 7) , (3 , 5) , (3 , 7)\}$
then $X = \dots\dots\dots$ $Y = \dots\dots\dots$
- 7) If $f(x) = 5x - 7$ then $f(3) = \dots\dots\dots$
- 8) If $f(x) = 6x$ then $f(2) + f(-2) = \dots\dots\dots$
- 9) Function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 3x$ represented by a straight line passes through the point $(-4 , \dots\dots\dots)$
- 10) The linear function $f(x) = x + 7$ is represented by a straight line cuts X – axis at the point
- 11) The linear function $f(x) = 2x - 1$ is represented by a straight line cuts y – axis at the point
- 12) If the point $(a , 3)$ lies on the straight line which represents the function $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = x - 5$ then $a = \dots\dots\dots$
- 13) If $f(x) = x - 6$ and $\frac{1}{3}f(a) = -2$ then $a = \dots\dots\dots$
- 14) If $X = \{1 , 3 , 5\}$, $f : X \rightarrow \mathbb{R}$ and $f(x) = 2x + 1$ then the range of $f = \dots\dots\dots$



- 15) The linear function $f(x) = 2 - 3x$ is represented by a straight line cuts Y – axis at the point
- 16) If f is function where $f : X \rightarrow Y$ then X is called and Y is called
- 17) If f is function from set X to set Y then the range of function $f \subset$
- 18) f is a function where $f(x) = 3x - 1$ is represented graphically by a line passes through point $(a, 2)$ then $a =$
- 19) $(2, -4) \in f(x)$ where $f(x) = kx + 8$ then $k =$

Choose the correct answer from those given:-

- 1) If $n(x^2) = 9$, then $n(X) =$
- a) 3 b) 6 c) 18 d) 81
- 2) The point $(-3, 4)$ lies in Quadrant.
- a) first b) second c) third d) fourth
- 3) $X = \{5, 6, 7\}$ then $n(X^2) =$
- a) 3 b) 6 c) 9 d) 12
- 4) of $X \times Y \{(1, 3), (1, 4)\}$ then $n(X) =$
- a) 3 b) 1 c) 4 d) 2
- 5) If $X = \{5\}$ $Y = \{3\}$ then $n(X \times Y)$
- a) 15 b) 8 c) 2 d) 1

6) If $X = \{3, 5, 7\}$ and R is a relation on X then the relation which represents a function is

- a) $R = \{(3, 5), (5, 3), (3, 7)\}$ b) $R = \{(3, 5), (5, 7)\}$
c) $R = \{(3, 5), (5, 5), (7, 5)\}$ d) $R = \{(3, 3), (3, 5), (3, 7)\}$

7) If the point $(x, 7)$ lies on $Y =$ axis then $5x + 1 =$

- a) zero b) 1 c) 5 d) 6

8) If R is a function from set X to set Y where

$X = \{2, 5, 8\}$, $Y = \{3, 5\}$ and $R = \{(2, 3), (5, 3), (x, 3)\}$ then $x =$

- a) 2 b) 3 c) 5 d) 8

9) If R is a function where $R = \{(4, 3), (5, 6), (9, 9)\}$ then the range of the function R is

- a) $\{3, 4, 5, 6, 9\}$ b) $\{4, 5, 9\}$
c) $\{3, 6, 9\}$ d) $\{3, 9\}$

10) If $f(x) = 7x - \frac{1}{2}$ then $f\left(\frac{1}{2}\right) =$

- a) 7 b) $\frac{1}{2}$ c) $\frac{7}{2}$ d) 3

11) If $f(x) = 4x + b$, $f(3) = 15$ then $b =$

- a) 156 b) 3 c) 4 d) -3

12) If $(m, 13)$ satisfies the function f where $f(x) = 3x + 4$ then $m =$

- a) 6 b) -6 c) 3 d) -3



13) If $(2, b)$ satisfies the function f where $f(x) = 3x - 6$, then $b = \dots\dots\dots$

- a) Zero b) 7 c) 9 d) 2

14) If $f(x) = x^2 + 7$ then $f(3) = \dots\dots\dots$

- a) 10 b) 7 c) 9 d) 16

15) If $f(x) = x^3$ then $f(2) + f(-2) = \dots\dots\dots$

- a) 16 b) Zero c) -7 d) 4

16) if $(2, -6)$ satisfies the function f where $f(x) = kx + 8$ then $k = \dots$

- a) -16 b) 7 c) -7 d) 2

17) The function f , where $f(x) = 5x$ is represented graphically by a straight line passes through the point.....

- a) $(5, 5)$ b) $(0, 0)$ c) $(0, 5)$ d) $(5, 0)$

18) If $f(x) = 5x + 4$ is represented graphically by a straight line passes through the point $(3, b)$ then $b = \dots\dots\dots$

- a) 5 b) 4 c) 3 d) 19

19) If the function f is a function from set X to set Y then the domain of the function is

- a) X b) Y c) $X \times Y$ d) $Y \times X$

Answer the following questions:-

- 1) If $X = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation on X where aRb means "a is twice b" for all $a, b \in X, a \neq b$.
 - a) Write R and represent it by an arrow diagram.
 - b) Is $(0, 0) \in R$?
 - c) Is $2R4$?
 - d) Find x If $6Rx$
- 2) If $X = \{2, 4, 8\}$, $Y = \{4, 6, 12, 24\}$, and R is a relation from X to Y such that aRb means " $b > 2a$ " for all $a \in X, b \in Y$, write R and represent it by an arrow diagram and by a Cartesian diagram.
- 3) If $X = \{13, 14, 43, 84\}$, and R is a relation on X such that aRb means "two numbers a and b have the same unit digit" for all $a, b, \in X$. write R and represent it on a lattice.
- 4) If $X = \{2, 3, 4, 7\}$, $Y = \{1, 2, 3, 4, 7, 8\}$ and R is a relation from X to Y where aRb means " $a - b$ is a prime number" for all $a \in X, b \in Y$ an arrow diagram.
- 5) If $X = \{0, 1, 2, 3\}$, $Y = \{-3, -2, -1, 0\}$ and R is a relation from X to Y where aRb means " a is additive –inverse of b " for all $a \in X, b \in Y$, write R and represent it by an arrow diagram and graphically. Is R a function? Why?
- 6) If $X = \{2, 5, 8\}$, $Y = \{10, 16, 24, 30\}$ and R is a relation from X to Y for all $a \in X, b \in Y$ where " a is factor of b " write R and represent it by an arrow diagram. Is R a function? Why?

- 7) If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where aRb means " $a + b = 7$ " for all $a \in X$, $b \in Y$, write and represent it by an arrow diagram and by a Cartesian diagram, show that R is a function? Write its domain and its range.
- 8) If $X = \{2, 3, 4\}$, $Y = \{3, 4, 5, 6, 7, 8\}$ and $f: X \rightarrow Y$ where $f(x) = 9 - x$ find the images of the elements of x and represent it by an arrow diagram.
- 9) If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$ then find the numerical value of the expression $a + b$.
- 10) Graph the function f where $f(x) = 4 - x^2$ in the interval $[-3, 3]$, from the graph determine:
First : The coordinates of the maximum value of function.
Second : The equation of the axis of symmetry.
- 11) Graph the function f , where $f(x) = x(6-x) + 4$ in the interval $[-1, 7]$
- 12) Represent the following linear functions graphically. $f(x) = 3x + 1$
- 13) If the straight line which represents the function $f: \mathbb{R} \rightarrow \mathbb{R}$, where $f(x) = 6x - a$ cuts Y - axis at the point $(b, 3)$ then find the value of a and b
- 14) If $X = \{3, 4, 5, 10, 13\}$, $Y = \{4, 5, 7, 8, 9, 19, 25\}$ and R is a relation from X to Y such aRb means " $b = 2a - 1$ " for all $a \in X$ and $b \in Y$.
- Write R
 - Represent R by a Cartesian diagram.
 - Find the value of x if $(x, 9) \in R$.



- 15) If $X = \{3, 5, 7, 9\}$, $Y = \{a : a \in \mathbb{N}, 10 \leq a < 50\}$ and R is relation from X to Y , where $R = \{(3, 15), (5, 25), (7, 35), (9, 45)\}$. Write the rule of R .
- 16) If $X = \{1, 2, 3\}$, $Y = \{1, 3, 6, 9, 13\}$ and R is a relation from X to Y where means " $a = \frac{1}{3}b$ " for all $a \in X, b \in Y$. write R and show that it is a function, write its range.
- 17) If function $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$.
- Write each of domain and range of f .
 - write the rule of the function f .
- 18) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is represented by a straight line cuts Y -axis at $(b, 3)$ where $f(x) = 6x - a$ find value of $2a + 7b$.
- 19) Find the value of x if $\frac{x+7}{x+11} = \frac{2}{3}$
- 20) Find the value of x if $(2x + 3) : (9x - 3) = 6 : 5$
- 21) Find the number which if added to the two terms of ratio $7 : 11$ it will be $2 : 3$
- 22) Two integers the ratio between them is $3 : 7$ and if subtracted 5 from each terms the ratio between them because $1 : 3$ find the two numbers.

Part Two

(1) Complete the following:-

- 1) If $3a = 4b$ then $a : b = \dots : \dots$
- 2) If 3, 4, c and 8 are proportional then $c = \dots$
- 3) The proportional mean of $3a^2$ and $27a^3b^2$ is \dots
- 4) If $\frac{a}{3} = \frac{y}{5}$, then $\frac{3a}{5y} = \dots$
- 5) If 9, 25, $2x$, $\frac{1}{y^2}$ are proportional quantities then $xy = \dots$
- 6) If $4x^2 - 12xy + 9y^2 = 0$ and $x \in \mathbb{R}, y \neq 0$ then $\frac{x}{y} = \frac{\dots}{\dots}$
- 7) If $\frac{a}{b} = \frac{2}{3}$ and $\frac{a}{c} = \frac{3}{5}$ then $a : b : c = \dots : \dots : \dots$
- 8) If $\frac{a}{b} = \frac{7}{2}$ then $\frac{a-b}{a+b} = \frac{\dots}{\dots}$
- 9) $\frac{x}{6} = \frac{y}{5} = \frac{z}{4} = \frac{\dots}{11} = \frac{2y+2}{\dots}$
- 10) If 1, x, 9, y are in continued proportion then $x = \dots$ $Y = \dots$

(2) Choose the correct answer from those given:

1- The third proportion of the two numbers 3 and 6 is \dots

- a) $\frac{1}{2}$ b) 2 c) 9 d) 12

2- If 2, 6, x + 15 are proportional then $x = \dots$

- a) 1 b) 2 c) 3 d) 4

3- If a, b, 2 and 3 are proportional, then $\frac{a}{b} = \dots$

- a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

4- If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and b 0) then $\frac{a}{b} = \dots\dots\dots$

- a) $\frac{2}{3}$ b) $\pm \frac{3}{2}$ c) $\pm \frac{2}{3}$ d) $\pm \frac{4}{3}$

5- The second proportion of the quantities $12ab^2$, , $21ab$, $14b^2$.

- a) $8ab^2$ b) $8b^3$ c) $24ab$ d) $24b^4$

6- If $\frac{x}{y} = \frac{z}{\ell}$ which of the following is true ?

- a) $\frac{x}{\ell} = \frac{y}{z}$ b) $\frac{x}{z} = \frac{\ell}{y}$ c) $\frac{x}{y} = \frac{\ell}{z}$ d) $\frac{x}{z} = \frac{y}{\ell}$

7- The number which added to each of the numbers 1,3,7,15 respectively to be in continued proportion is

- a) 1 b) 2 c) 3 d) 4

8- If $\frac{a}{2} = \frac{b}{3}$ then $\frac{b-a}{b+a}$ equals

- a) $\frac{1}{5}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$

9- If $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$, then z =

- a) -2 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) 2

10- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ (where $m \in \mathbb{R}^*$), then $\frac{a+c+e}{b+d+f}$ equals

- a) m b) 3m c) m^3 d) $3m^3$

(3) If the following sets of numbers are proportional, then find the values of x.

- a) 8, x, 4, 5 b) 11,3,x,6 c) 6, 24, 1, x

(4) Find. x : y : z in each of the following.

- a) $\frac{x}{y} = \frac{3}{5}$ and $\frac{y}{z} = \frac{4}{7}$ b) $\frac{x}{y} = \frac{4}{5}$ and $\frac{x}{z} = \frac{3}{7}$

(5) If $\frac{a}{b} = \frac{2}{5}$, then find the value of each of the following ratios.

a) $\frac{a+b}{b}$

b) $\frac{a}{b-a}$

c) $\frac{b-a}{b+a}$

d) $\frac{7a-2b}{3a+2b}$

(6) If $\frac{a}{2} = \frac{b}{3} = \frac{2a-b}{m}$, then find the value of m

(7) If $\frac{a}{b-a} = \frac{c}{d-c}$, then prove that a , b, c and d are proportional.

(8) If b is the middle proportional between a and c, then prove that.

a) $\frac{a^2}{b^2} = \frac{b^2}{c^2} = \frac{a}{c}$

b) $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = b^2$

(9) If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that :

a) $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$

b) $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

(10) If a -1, a + 1 , b -2 , b +2 are in proportion, then find $\frac{a}{b}$, then prove

that $\frac{a+b}{a+b-3} = \frac{3a}{5a-b-3}$

(11) If $\frac{a}{b} = \frac{1}{3}$, $\frac{a}{c} = \frac{1}{9}$ and a + b + c = 26, then find each of a, b and c.

(12) If x , y , z, ℓ are proportional quantities , then prove that:

a) $\frac{x+y}{z+\ell} = \frac{2x^2-3y^2}{2z^2-3\ell^2}$

b) $\sqrt[3]{\frac{5x^2-3y^2}{5y^3-3\ell^3}} = \frac{x+z}{y+\ell}$

(13) If $\frac{x+y}{\ell+m} = \frac{y+z}{m+n} = \frac{z+x}{n+\ell}$, then prove that : $\frac{x}{\ell} = \frac{y-x}{m-\ell}$

(14) If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, then prove that : $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

(15) If $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$, then prove that : $\frac{x+y+z}{x-z} = 5$

(16) Find the number that should be added to each of the numbers 7, 9, 12 , 15 to be proportional.



- (17) Two positive integer numbers, the ratio between them is 3 : 7 and if we subtract 5 from each of them the ratio becomes 1 : 3, find the two numbers.
- (18) Find the positive number that if we add its square to each term of the ratio 7 : 11 it becomes 4 : 5

Exercises

(1) Complete the following:

- 1- If $y = 3x$ then $y \propto \dots\dots\dots$
- 2- If $x - y - 7 = 0$ then $y \propto \dots\dots\dots$
- 3- If $y \propto x$ and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$
- 4- If $y \propto x$ and $x = 1$ as $y = 4$ then the constant of variation is $\dots\dots\dots$
- 5- If $y \propto x$ and $y = 2$ when $x = 4$ then $y = \dots\dots\dots x$
- 6- If y varies inversely as x and $y = 2$ when $x = \frac{1}{2}$ then $y = \frac{\dots\dots}{x}$
- 7- If $x^2y^2 - 4xy + 4 = 0$ then $y \propto \dots\dots\dots$
- 8- If $y^2 - 6xy + 9x^2 = 0$ then $y \propto \dots\dots\dots$
- 9- If $y \propto \frac{1}{x}$ and the variable x took the two values x_1 and x_2 and the variable y took the two values, y_1 and y_2 respectively then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$
- 10- If $y \propto x$ and $y = 1$ when $x = 4$ then $y = \dots\dots\dots$ when $x=8$

(2) Choose the correct answer from those given:

1- The relation which represents direct variation between the two variables x and y is

- a) $x y = 7$ b) $y=x+2$ c) $\frac{x}{3} = \frac{4}{y}$ d) $\frac{x}{5} = \frac{y}{2}$

2- If y varies inversely as x and if $x = \sqrt{3}$ as $y = \frac{2}{\sqrt{3}}$, then the constant of variation equals.

- a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 2 d) 6

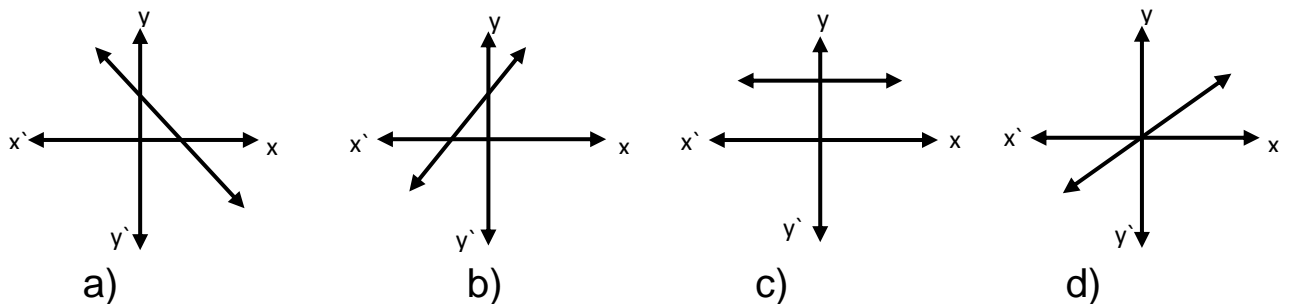
3- If $y-x = \frac{1}{x} - \frac{1}{y}$ where $x \neq Y \neq 0$ then

- a) $y \propto x + 1$ b) $y \propto x$ c) $y \propto \frac{1}{x}$ d) $y \propto \frac{1}{x^2}$

4- If the total cost (y) for a certain trip and if the some of this total cost is constant (a) and the other changes with the number of participated (x), which of the following relations is correct:

- a) $y = a x$ b) $y = \frac{a}{x}$
 c) $y = a + \frac{m}{x}$, (m is a constant $\neq 0$)
 d) $y = a + mx$, (m is a constant $\neq 0$)

5- The graph which represents the direct variation between x and y is



(3) Show which of following tables represent direct variation, inverse variation or neither –nor, state the reason in each case:

X	Y
3	20
5	12
4	15
6	10

X	Y
2	9
4	18
6	54
16	72

X	Y
5	9
10	18
15	27
25	45

X	Y
3	6
-2	-9
-18	1
9	-2

- (4) If y varies as x and y = 10 when x = 7 , find x when y = 20
- (5) If y varies inversely as x and y = 10 when x=3, find y when x=5
- (6) If $y \propto x$ and y = 20 when x =7 find the relation between x and y then find the value of y when x = 14
- (7) If $y \propto \frac{1}{x}$ and $x = 2\frac{4}{5}$ when $y = \frac{4}{7}$, then find the relation between x and y then find also the value of y when $x = 3\frac{1}{5}$
- (8) If $y = 3 + a$ and $a \propto \frac{1}{x}$ if y = 5 when x =1, then find the relation between x and y and find y when x = 2.
- (9) Let $y = a + 7$ and $a \propto \frac{1}{x^2}$ if a = 18 when $x = \frac{2}{3}$ find the relation between y and x, then deduce the value of y when x = 6 .
- (10) If $\frac{21x-y}{7x-z} = \frac{y}{z}$, then prove that $y \propto z$
- (11) From the data of the following table answer the following questions.

X	2	4	6
Y	6	3	2

- a) Identify the kind of variation whether it is direct or inverse.
- b) Find the constant of variation.
- c) Find the value of x when $y = 2\frac{2}{5}$



- 12) A car moves with constant velocity such that the covered distance varies directly as the time if the car covered 90 km within one and half an hour. Write down the relation between the covered distance and the time, then find the covered distance within $2\frac{1}{2}$ hours.
- 13) If the light tension (t) of a lamp varies inversely as the square of the distance (d) between the lamp and a pupil studies his lessons at a distance of 12 metres. If the tension of light is weak, then what is the distance which the lamp should be far from the pupil in order that the light tension becomes 4 times what it was before?
- 14) If the height of a right circular cylinder (h) of a constant volume varies inversely as the square of the radius length of its base (r) and $h = 18\text{cm}$ when $r = 7\text{cm}$. find the height h when $r = 10.5\text{ cm}$.
- 15) A car of mass 3 ton moves with uniform velocity under resistance varies as its velocity. If the resistance was 6 kg. weight/ton of the mass of the car when the velocity was 50 km/H find the velocity of the car if the resistance becomes 27 kg. weight/ton .

Problem on statistics

(Collecting data and dispersion)

First : Complete the following:-

- 1- The statistical sample is a part of
- 2- From the means of collecting data are and
- 3- The arithmetic mean is one of measure of while' the range is one measures of
- 4- The difference between the greatest value and the smallest value of a set of data is
- 5- The arithmetic mean of a set of values of individuals equals
- 6- The positive square root of the squares deviations of values from its arithmetic mean is called
- 7- The range of the set of values 5 , 14 , 4 , 21, 16 and 12 is
- 8- The difference between the greatest individual and the smallest individual of a set of values is called.....
- 9- If 78 is the greatest individual of a set of individuals and its range is 39 then the smallest individual of this set equals

Second: Choose the correct answer from those given:-

- 1- Selecting a sample of layers of a statistical society is called sample.
a) random b) class (payer) c) deliberate d) bunch
- 2- The difference between the greatest value and the smallest value of a set of individuals is called
a) the range b) the arithmetic mean
c) the median d) the standard deviation



- 3- The range of the set of the values 7 , 3 , 6 , 9 and 5 equals.
a) 3 b) 4 c) 6 d) 12
- 4- The arithmetic mean of the set of the values 7 , 3 , 6 , 9 and 5 equals.....
a) 3 b) 4 c) 6 d) 12
- 5- If $\sum(x - \bar{x})^2 = 36$ for a set of values whose number is 9 then $\sigma = \dots$
a) 2 b) 4 c) 18 d) 27

Third:

- 1- Calculate the arithmetic mean and the standard deviation of the set of values: 16 , 32 , 5 , 20 and 27
- 2- Calculate the standard deviation for the values:3 ,12,17 , 28 , 30.
- 3- Calculate the mean and standard deviation to the following data 12 ,13,16 , 18 and 21.
- 4- Calculate the arithmetic mean and the standard deviation of the set of values 73 , 54 , 62 , 71 , 60
- 5- The following table represents the number of children of 100 families in a city.

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate each of the arithmetic mean and standard deviation.

- 6- The following table represents the number of children of 26 families in a city.

Number of children	0	1	2	3	4	5	Total
Number of families	9	1	6	3	5	2	26

Calculate the standard deviation.

- 7- Find the arithmetic mean and standard deviation of the following data:

The set	0-	2-	4-	6-	8-
The frequency	5	9	15	15	6

Model Answers

Part (1)

Complete each of the following:-

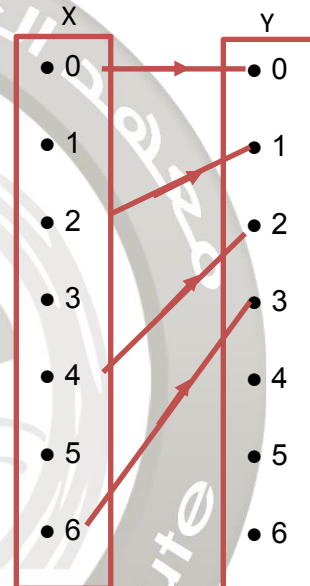
- 1) 4th
- 2) $X = -4$, $Y = 2$
- 3) $n(y) = 3$
- 4) X – axis
- 5) $X + Y = 6$
- 6) $X = \{1, 2, 3\}$, $Y = \{5, 7\}$
- 7) $f(3) = 8$
- 8) Zero.
- 9) -12
- 10) (-7 , 0)
- 11) (0 , -1)
- 12) $a = 8$
- 13) $a = \text{zero}$.
- 14) Range = { 3, 7 11 }.
- 15) (0 , 2)
- 16) Domain , Codomain.
- 17) Y
- 18) $a = 1$
- 19) $K = -6$

Choose the correct answer from those given:-

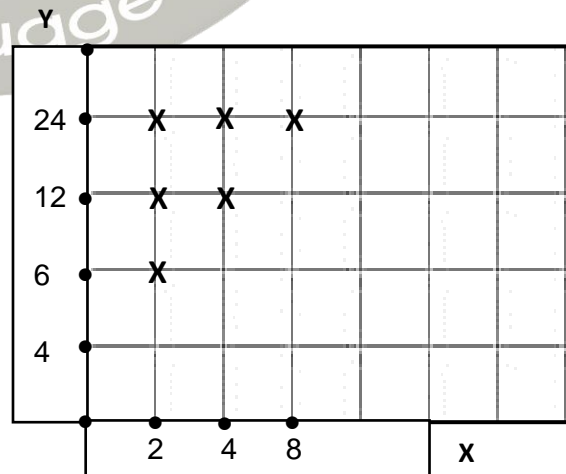
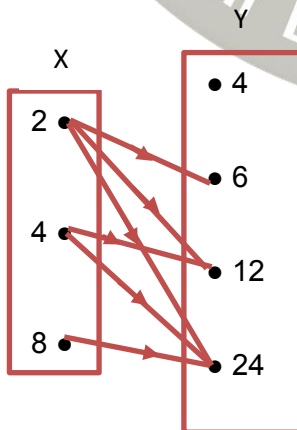
- | | | | |
|-------|-------|-------|-------|
| 1) a | 2) b | 3) c | 4) b |
| 5) d | 6) c | 7) b | 8) d |
| 9) c | 10) d | 11) b | 12) c |
| 13) a | 14) d | 15) b | 16) c |
| 17) b | 18) d | 19) a | |

Answer the following questions:-

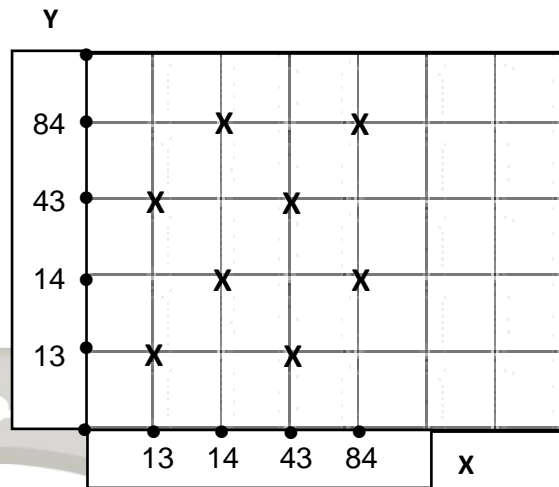
- 1) a) $R = \{ (0, 0), (4, 2), (6, 3), (2, 1) \}$
 b) $(0, 0) \in R$
 c) $(2, 4) \notin R$
 d) $(x = 3)$



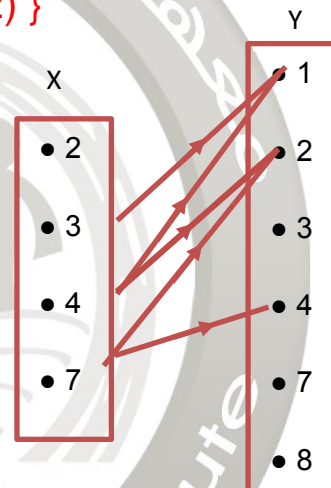
- 2) $R = \{ (2, 6), (2, 12), (2, 24), (4, 12), (4, 24), (8, 24) \}$



3) $R = \{ (13, 13), (13, 43), (14, 14), (14, 84), (43, 13), (43, 43), (84, 14), (84, 84) \}$.



4) $R = \{ (3, 1), (4, 1), (7, 2), (7, 4), (4, 2) \}$



5) $R = \{ (0, 0), (1, -1), (2, -2), (3, -3) \}$, yes because each element in x connected to only one element from y
 - Represent by yourself.

6) $R = \{ (2, 10), (2, 16), (2, 24), (2, 30), (5, 10), (5, 30), (8, 16), (8, 24) \}$, No because $2 \in x$ appeared many times as 1st projection in the relation.

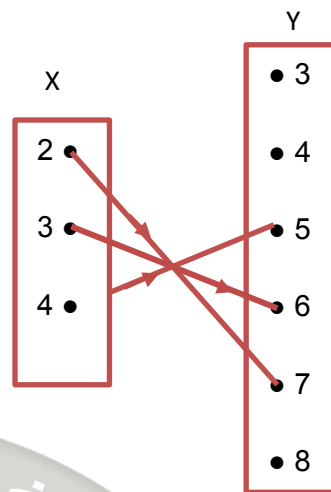
7) $R = \{ (1, 6), (3, 4), (4, 3), (5, 2) \}$, R is a function from $X \rightarrow Y$
 range = $\{ 6, 4, 3, 2 \}$
 domain = $\{ 1, 3, 4, 5 \}$
 - Represent by yourself

8) $f(2) = 9 - 2 = 7$

$f(3) = 9 - 3 = 6$

$f(4) = 9 - 4 = 5$

$f = \{ (2, 7), (3, 6), (4, 5) \}$



9) $a = 5$ or 3 , $b = 3$ or 5

$\therefore a + b = 8$

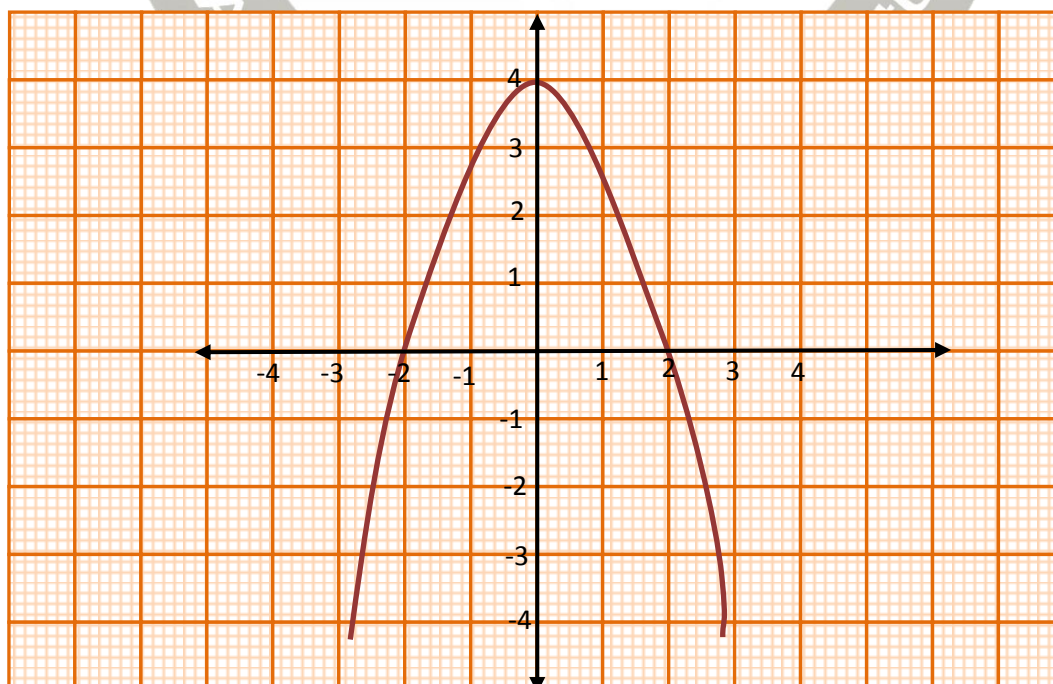
10) $f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
f(x)	-5	0	3	4	3	0	-5

Co-ordinate of max value (0 , 4)

Equation of the axis of symmetry $x = 0$

Maximum value = 4



11) $f(x) = x(6 - x) + 4 = 6x - x^2 + 4$ (solve by yourself)

12) $f(x) = 3x + 1$

a) $f(0) = 0 + 1 = -1$

b) $f(1) = 3 + 1 = 4$

c) $f(-1) = -3 + 1 = -2$

represent it by you self.

x	-1	0	1
f(x)	-2	1	4

13) $f(x) = 6x - a$

∴ The line cuts y-axis at (b, 3), then $b = 0$ (0, 3) satisfies the equation of the line.

∴ $f(0) = 6 \times (0) - a = 3 \rightarrow -a = 3 \rightarrow a = -3$

14) a) $R = \{(3, 5), (4, 7), (5, 9), (10, 19), (13, 25)\}$

b) solve by yourself

c) ∴ $9 = 2x - 1 \Rightarrow 2x = 10 \Rightarrow x = 5$

15) ∴ $R = \{(3, 15), (5, 25), (7, 35), (9, 45)\}$

\downarrow \downarrow \downarrow \downarrow
 $3 \times 5 = 15$ $5 \times 5 = 25$ $7 \times 5 = 35$ $9 \times 5 = 45$

The rule of R is $b = 5a$

16) $R = \{(1, 3), (2, 6), (3, 9)\}$, it's a function, range = $\{3, 6, 9\}$.

17) a) domain = {1 , 2 , 3 , 4 , 5} , Range = {3 , 5 , 7 , 9 , 11}

(1 , 3)	(2 , 5)	(3 , 7)	(4 , 9)	(5 , 11)
↓	↓	↓	↓	↓
$1 \times 2 + 1 = 3$	$2 \times 2 + 1 = 5$	$3 \times 2 + 1 = 7$	$4 \times 2 + 1 = 9$	$5 \times 2 + 1 = 11$

The rule of the function is $2a + 1$

18) $b = 0$, $f(x) = 6x - a \Rightarrow 3 = 6 \times (0) - a$

$a = -3$ the value of $2a + 7b = 2(-3) + 7(0) = -6$

19) $3(x+7) = 2(x+11)$

$$3x + 21 = 2x + 22$$

$$X = 1$$

20) $10X + 15 = 54X - 18$

$$44X = 33 \quad \rightarrow X = \frac{3}{4}$$

21) $\frac{7+x}{11+x} = \frac{2}{3}$

$$21 + 3X = 22 + 2X$$

$$X = 1$$

22) let the first no $3X$, let the second number $7X$

$$\frac{3x-5}{7x-5} = \frac{1}{3}$$

$$9X - 15 = 7X - 5$$

$$X = 5$$

The first no. = $3 \times 5 = 15$

The second no. = $7 \times 5 = 35$

Part Two

(1) Complete each of the following:-

1) $a : b = 4 : 3$

2) $\frac{3}{4} = \frac{c}{8} \Rightarrow c = \frac{8 \times 3}{4} = 6$

3) Proportional mean = $\pm \sqrt{1^{st} \times 3^{rd}}$
 $= \pm \sqrt{3ab^2 \times 27a^3b^2} =$
 $= \pm \sqrt{81a^2 \times b^4} = \pm 9a^2b^2$

4) $\frac{3x}{5y} = \frac{9}{25}$

5) $\frac{9}{25} = \frac{x^2}{\frac{1}{y^2}}$

$\frac{9}{25} = x^2y^2 \Rightarrow xy = \pm \frac{3}{5}$

6) $4x^2 - 12xy + 9y^2 = 0$

$(2x - 3y)^2 = 0 \Rightarrow 2x - 3y = 0$

$2x = 3y \Rightarrow \frac{x}{y} = \frac{3}{2}$

7) $\frac{a}{b} = \frac{2}{3}, \frac{a}{c} = \frac{3}{5}$

$a : b : c$

$\boxed{2} : 3 : 2$

$\boxed{3} : 3 : 5$

$6 : 9 : 10$

8) $a = 7m, b = 2m \rightarrow \frac{7m-2m}{7m+2m} = \frac{5m}{9m} = \frac{5}{9}$

9) $\frac{x+y}{11} = \frac{2y+z}{14}$

$$10) \frac{1}{x} = \frac{x}{9} = \frac{9}{y}$$

$$x^2 = 9 \Rightarrow x = \pm \sqrt{9} = \pm 3$$

$$\Rightarrow yx = 81$$

$$y = \frac{81}{\pm 3} = \pm 27$$

(2) Choose: -

1) d

2) c

3) a

4) b

5) b

6) d

7) a

8) a

9) d

10) a

Steps for 7, 9

$$7) \frac{1+x}{3+x} = \frac{3+x}{7+x} = \frac{7+x}{15+x}$$

$$(3+x)(3+x) = (1+x)(7+x)$$

$$x^2 + 6x + 9 = x^2 + 8x + 7$$

$$8x - 6x + 9 - 7 \rightarrow 2x = 2$$

$$x = 1$$

$$9) \frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$$

$$z = 4 \times 2 - 2 \times 3$$

$$z = 8 - 6 = 2$$

(3)

$$a) \frac{8}{x} = \frac{4}{5} \rightarrow x = 10$$

$$b) \frac{11}{3} = \frac{x}{6} \rightarrow x = 22$$

$$c) \frac{6}{24} = \frac{1}{x} \rightarrow x = 4$$

(4) a)

$$x : y : z$$

$$3 : 5 : 5$$

$$4 : 4 : 7$$

$$12 : 20 : 35$$

b)

$$x : y : z$$

$$4 : 5 : 4$$

$$3 : 3 : 7$$

$$12 : 15 : 28$$

(5) a) $\frac{a+b}{a} = \frac{2m+5m}{2m} = \frac{7}{2}$

b) $\frac{a}{b-a} = \frac{2m}{5m-2m} = \frac{2}{3}$

c) $\frac{b-a}{b+a} = \frac{7m-2m}{7m+2m} = \frac{5}{9}$

d) $\frac{7a-2b}{3a+2b} = \frac{14m-10m}{6m+10m} = \frac{1}{4}$

(6) $\frac{2a-b}{m} = \frac{2a-b}{4-3} \Rightarrow m = 1$

(7) $\frac{a}{b-a} = \frac{c}{d-c} \quad \therefore ad - ac = bc - ac$

$$\therefore ad = bc$$

$$\therefore \frac{a}{b} = \frac{c}{d} \quad \therefore a, b, c, d \text{ are prop.}$$

(8) $\frac{a}{b} = \frac{b}{c} = m \quad \therefore a = cm^2, b = cm$

a) L.H.S. = $\frac{a^2}{b^2} = \frac{c^2m^4}{c^2m^2} = m^2$ _____ (1)

M.H.S. = $\frac{b^2}{c^2} = \frac{c^2m^2}{c^2} = m^2$ _____ (2)

R.H.S. = $\frac{a}{c} = \frac{cm^2}{c} = m^2$ _____ (3)

$$\begin{aligned}
 \text{b) } & \frac{a + b + c}{a^{-1} + b^{-1} + c^{-1}} \\
 & = \frac{cm^2 + cm + c}{c^{-1}m^{-2} + c^{-1}m^{-1} + c^{-1}} \\
 & = \frac{c(m^2 + m + 1)}{c^{-1}m^{-2}(1 + m + m^2)} \\
 & = c^2m^2 \rightarrow (1) \\
 & = b^2 = c^2m^2 \rightarrow (2)
 \end{aligned}$$

From (1) & (2)

L.H.S = R.H.S

(9) $x = 3k, y = 4k, z = 5k$

a) $\frac{2y - z}{3x - 2y + z} = \frac{8k - 5k}{9k - 8k + 5k} = \frac{3k}{6k} = \frac{1}{2}$

b) $\sqrt{3x^2 + 3y^2 + z^2}$ L.H.S.

$$= \sqrt{3(3k)^2 + 3(4k)^2 + (5k)^2}$$

$$= \sqrt{27k^2 + 48k^2 + 25k^2}$$

$$= \sqrt{100k^2} = 10k$$

$$2x + y = 6k + 4k = 10k \quad \text{R.H.S.}$$

L.H.S. = R. H. S.

(10) $\frac{a-1}{a+1} = \frac{b-2}{b+2}$

$$(a - 1)(b + 2) = (a + 1)(b - 2)$$

$$ab + 2a - b - 2 = ab - 2a + b - 2$$

$$2a - b = -2a + b$$

$$2a + 2a = b + b$$

$$4a = 2b \rightarrow \frac{a}{b} = \frac{2}{4} = \frac{1}{2}$$

$$a = m, b = 2m$$

$$\begin{aligned} \text{L.H.S.} &= \frac{a+b}{a+b-3} = \frac{m+2m}{m+2m-3} = \frac{3m}{3m-3} \\ &= \frac{3m}{3(m-1)} = \frac{m}{m-1} \end{aligned} \quad \text{_____ (1)}$$

$$\begin{aligned} \text{R.H.S.} &= \frac{3a}{5a-b-3} = \frac{3m}{5m-2m-3} = \frac{3m}{3m-3} \\ &= \frac{3m}{3(m-1)} = \frac{m}{m-1} \end{aligned} \quad \text{_____ (2)}$$

From (1) and (2) L.H.S. = R. H. S.

(11) $\frac{a}{b} = \frac{1}{3}, \frac{a}{c} = \frac{1}{9}, a + b + c = 26$

$$a = m, b = 3m, c = 9m$$

$$m + 3m + 9m = 26 \rightarrow 13m = 26$$

$$m=2 \rightarrow a = 2, b=6, c=18$$

(12) $\frac{x}{y} = \frac{z}{\ell} = m$

$$x = ym, z = \ell m$$

$$\begin{aligned} \text{L.H.S.} &= \left(\frac{x+y}{z+\ell}\right)^2 = \left(\frac{ym+y}{\ell m+\ell}\right)^2 = \left(\frac{y(m+1)}{\ell(m+1)}\right)^2 \\ &= \left(\frac{y}{\ell}\right)^2 = \frac{y^2}{\ell^2} \end{aligned} \quad \text{_____ (1)}$$

$$\begin{aligned} \text{R.H.S.} &= \frac{2x^2-3y^2}{2z^2-3\ell^2} = \frac{2y^2 m^2-3y^2}{2\ell^2 m^2-3\ell^2} \\ &= \frac{y^2 (2m^2-3)}{\ell^2 (2m^2-3)} = \frac{y^2}{\ell^2} \end{aligned} \quad \text{_____ (2)}$$

From (1) and (2) L.H.S. = R.H.S.

$$\begin{aligned}
 \text{b) L.H.S.} &= \sqrt[3]{\frac{5x^3-3z^3}{5y^3-3\ell^3}} = \\
 &= \sqrt[3]{\frac{5y^3 m^3-3\ell^3 m^3}{5y^3-3\ell^3}} = \sqrt[3]{\frac{m^3(5y^3-3\ell^3)}{5y^3-3\ell^3}} \\
 &= \sqrt[3]{m^3} = m \quad \text{_____ (1)}
 \end{aligned}$$

$$\text{R.H.S.} = \frac{x+z}{y+\ell} = \frac{ym+\ell m}{y+\ell} = \frac{m(y+\ell)}{(y+\ell)} = m \quad \text{_____ (2)}$$

From (1) , (2) we get L.H.S. = R.H.S

(13) 1st - 2nd + 3rd = One of the ratios

$$\begin{aligned}
 \frac{x+y-y-z+z+x}{\ell+m-m-n+n+\ell} &= \frac{2x}{2\ell} = \frac{x}{\ell} \\
 &= \text{one of the ratios} \quad \text{(1)}
 \end{aligned}$$

2nd - 3rd = One of the ratios

$$\frac{y+z-z-x}{m+n-n-\ell} = \frac{y-x}{m-\ell} = \text{one of the ratios} \quad \text{(2)}$$

From (1) and (2) we get

$$\frac{x}{\ell} = \frac{y-x}{m-\ell}$$

(14) 2x 1st + 2nd

$$\frac{2x+y}{4a+2b+2b-c} = \frac{2x+y}{4a+4b-c} = \text{One of the given ratios.} \quad \text{_____ (1)}$$

2x 1st + 2x 2nd + 3rd

$$\begin{aligned}
 \frac{2x+2y+z}{4a+2b+4b-2c+2c-a} &= \text{one of the given ratios} \\
 &= \frac{2x+2y+z}{3a+6b} = \text{one of ratios} \quad \text{_____ (2)}
 \end{aligned}$$

From (1) and (2) L.H.S. = R. H. S.

(15) By adding 1st + 2nd + 3rd

$$\frac{x+y+y+z+z+x}{7+5+8} = \frac{2x+2y+2z}{20}$$

$$= \frac{2(x+y+z)}{20} = \frac{x+y+z}{10} = \text{one of the given ratios} \quad \underline{\hspace{2cm}} \quad (1)$$

$x - z$ 1st - 2nd

$$\frac{x+y-y-z}{7-5} = \frac{x-z}{2} = \text{one of the given ratios} \quad \underline{\hspace{2cm}} \quad (2)$$

From (1) and (2) $\frac{x+y+z}{10} = \frac{x-z}{2} =$

$$\frac{x+y+z}{x-z} = \frac{10}{2} = 5$$

(16) $\frac{7+x}{9+x} = \frac{12+x}{15+x}$

$$(x + 15)(x + 7) = (x + 12)(x + 9)$$

$$x^2 + 7x + 15x + 105 = x^2 + 9x + 12x + 108$$

$$22x + 105 = 21x + 108 +$$

$$22x - 21x = 108 - 105$$

$$x = 3$$

(17) let the + ve. integers are x , y

$$\frac{x}{y} = \frac{3}{7} \rightarrow x = 3m, y=7m$$

$$\frac{3m-5}{7m-5} = \frac{1}{3}$$

$$9m-15 = 7m - 5$$

$$9m - 7m = 15 - 5$$

$$2m = 10 \Rightarrow m = 5$$

(18) let the no. is x

$$\frac{x^2+7}{x^2+11} = \frac{4}{5}$$

$$5x^2 + 35 = 4x^2 + 44$$

$$5x^2 - 4x^2 = 44 - 35$$

$$x^2 = 9 \Rightarrow x = \pm \sqrt{9} = \pm 3$$

Exercises

(1) Complete the following:

- 1) x 2) $\frac{1}{x}$ 3) $\frac{y_1}{y_2}$ 4) m = 4
 5) $\frac{1}{2}x$ 6) $\frac{1}{x}$ 7) $\frac{1}{x}$ 8) x
 9) $\frac{y_2}{y_1}$ 10) 2

(2) Choose

- 1) d 2) c 3) c 4) d 5) d

(3)

- The first table represents an inverse variation because $3 \times 20 = 5 \times 12 = 4 \times 15 = 6 \times 10 = 60$
 $\therefore xy = m$

x	y
3	20
4	15
5	12
6	10

- The 2nd table does not represent a direct or inverse variation because

$$2 \times 9 = 18, 4 \times 18 = 72 \text{ or } \frac{4}{18} \neq \frac{6}{54}$$

- The third table represent a direct variation

$$\text{because } \frac{9}{5} = \frac{18}{10} = \frac{27}{15} = \frac{45}{25} \therefore \frac{y}{x} = m$$

- The fourth table does not represent a direct variation

$$\text{or inverse because } 3 \times 6 \neq -18 \times 1 \text{ or } \frac{6}{3} \neq \frac{-9}{-2}$$

x	y
5	9
10	18
15	27
25	45

(4) $y \propto x \quad \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$$\frac{10}{20} = \frac{7}{x}$$

$$x = 7 \times 20 \div 10 = 14$$

(5) $y \propto \frac{1}{x} \quad \frac{y_1}{y_2} = \frac{x_2}{x_1}$

$$\frac{10}{y} = \frac{5}{3}$$

$$y = 3 \times 10 \div 5 = 6$$

(6) $y = mx \rightarrow 20 = 7m$

$$m = \frac{20}{7} \rightarrow y = \frac{20}{7}x$$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2} \rightarrow \frac{20}{y} = \frac{7}{14}$$

$$y = 14 \times 20 \div 7 = 40$$

(7) $\therefore y \propto \frac{1}{x}$

$$\therefore \frac{y_1}{y_2} = \frac{x_2}{x_1}$$

$$\frac{\frac{4}{7}}{y} = \frac{3\frac{1}{5}}{2\frac{5}{5}}$$

$$y = 2\frac{4}{5} \times \frac{4}{7} \div 3\frac{1}{5} = \frac{14}{5} \times \frac{4}{7} \div \frac{16}{5} =$$

$$y = \frac{14}{5} \times \frac{4}{7} \times \frac{5}{16} = \frac{1}{2}$$

(8) $y = 3 + a, a \propto \frac{1}{x}$

at $y = 5 \rightarrow 5 = 3 + a_1$

$$\rightarrow a_1 = 2$$

$$\frac{a_1}{a_2} = \frac{x_2}{x_1}$$

$$\begin{cases} y_1 = 5 \\ x_1 = 1 \\ y_2 = ? \\ x_2 = 2 \end{cases}$$

$$\frac{2}{a_2} = \frac{2}{1} \Rightarrow a_2 = 1 \times 2 \div 2 = 1$$

$$\text{at } a_2 = 1 \Rightarrow y_2 = 3 + 1 = 4$$

(9) $y = a + 7$ and $a \propto \frac{1}{x^2}$ { $a = 18$

$$a = \frac{m}{x^2} \rightarrow m = ax^2$$
{ $x_1 = \frac{2}{3}$

$$m = 18 \times \frac{4}{9} = 8$$
{ $y = ?$

$$a = \frac{8}{x^2}$$
{ $x = 6$

$$a = \frac{8}{(6)^2} = 8 \div 36 = \frac{2}{9}$$

$$\text{at } a = \frac{2}{9} \rightarrow y = \frac{2}{9} + 7 = 7\frac{2}{9}$$

Another solution for no. (5) $y \propto \frac{1}{x}$

$$\therefore y = \frac{m}{x} \quad \therefore 10 = \frac{m}{3} \quad \therefore m = 30$$

$$\therefore y = \frac{30}{x}$$

$$\therefore y = \frac{30}{5} \quad \therefore y = 6$$

Another solution for no. (8)

No. 8 $y = a + 3$

$$\therefore a \propto \frac{1}{x} \quad \therefore a = \frac{m}{x}$$

$$\therefore y = \frac{m}{x} + 3$$

$$\therefore 5 = m + 3 \quad \therefore m = 2$$

$$\therefore y = \frac{2}{x} + 3$$

$$\therefore y = \frac{2}{2} + 3 \quad \therefore y = 4$$

Another solution for no. (9)

No. 9 $y = a + 7$

$$\therefore a \propto \frac{1}{x^2}$$

$$\therefore a = \frac{m}{x^2}$$

$$\therefore 18 = \frac{9m}{4}$$

$$\therefore m = \frac{4 \times 18}{9} = 8$$

$$\therefore y = \frac{8}{x^2} + 7$$

$$\therefore y = \frac{8}{36} + 7 = \frac{65}{9}$$

(10) $\frac{21x-y}{7x-z} = \frac{y}{z}$

$$z(21x - y) = y(7x - z)$$

$$21xz - yz = 7xy - yz$$

$$21xz = 7xy$$

$$3z = y \rightarrow \therefore y = mz$$

$$\therefore y \propto z$$

(11) a) inverse variation

b) $\therefore y \propto \frac{1}{x}$

$$\therefore m = xy = 2 \times 6 = 12$$

c) $y = \frac{12}{x}$

at $x = 3$

$$y = \frac{12}{3} = 4$$

d) at $y = 2\frac{2}{5} = \frac{12}{5}$

$$x = 12 \div \frac{12}{5} = 5$$

(12) $d_1 = 90 \text{ km}$ $t_1 = 1\frac{1}{2} \text{ h}$
 $d_2 = ?$ $t_2 = 2\frac{1}{2} \text{ h}$
 $d \propto t \rightarrow d = mt$
 $90 = 1\frac{1}{2} m \rightarrow m = 90 \div \frac{3}{2} = 60$
 $d = 60t$
 $d = 60 \times 2\frac{1}{2} = 60 \times \frac{5}{2} = 150 \text{ km.}$

(13) $t \propto \frac{1}{d^2}$ $d_1 = 12$
 $t = \frac{m}{d^2}$ $t_1 = x$
 $x = \frac{m}{12^2}$ $d_2 = ?$
 $m = 12x = 144x$ $t_2 = 4x$
at $t = 4x$ $4x = \frac{12x}{d^2}$
 $d^2 = \frac{144}{4} = 36$
 $d = \sqrt{36} = 6$

Another solution

t_1 $d_1 = 12$
 $t_2 = 4t_1$ $d_2 = ?$
 $\therefore t \propto \frac{1}{d^2}$
 $\frac{t_1}{t_2} = \frac{d_2^2}{d_1^2}$
 $\frac{t_1}{4t_1} = \frac{d_2^2}{144}$
 $d_2^2 = \frac{144}{4} = 36 \rightarrow d_2 = \sqrt{36} = 6m$

$$(14) \quad h \propto \frac{1}{r_2}$$

$$\frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$$

$$\frac{18}{h_2} = \frac{110.25}{49}$$

$$h_2 = \frac{49 \times 18}{110.25} = 8$$

$$(15) \quad R \propto v \Rightarrow R = mv$$

$$\frac{R_1}{R_2} = \frac{v_1}{v_2}$$

$$\frac{6}{27} = \frac{50}{V_2}$$

$$V_2 = \frac{50 \times 27}{6} = 225 \text{ km / H}$$

Problem on statistics

First: Complete

- 1) Statistical society
- 2) Mass population, samples.
- 3) Central tendency, dispersion.
- 4) Range
- 5) $\frac{\text{sum values}}{\text{no of values}}$
- 6) Standard deviation
- 7) 17
- 8) Range
- 9) 39

Second: Choose

1) b

2) a

3) c

4) c

5) a

Third:

1) Form the tables by yourself , then : the man $\bar{X} = 20$, $\sigma = 9.32$

2) Form the tables by yourself , then : $\sigma = 10.06$

3) The mean $\bar{X} = \frac{12+13+16+18+21}{5} = 16$

, form the tables by yourself

, then : $\sigma = 3.29$

4) The mean $\bar{X} = \frac{73+54+62+71+60}{5} = 64$

, form the tables by yourself

, then: $\sigma = 7.07$

5) Form the tables by yourself

, then the mean $\bar{X} = 2.26$, $\sigma = 1.06$

6) Form the tables by yourself , then : $\sigma = 1.73$

7) Form the tables by yourself

, then the mean $\bar{X} = 5.32$, $\sigma = 2.31$